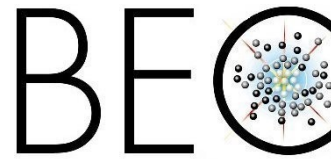


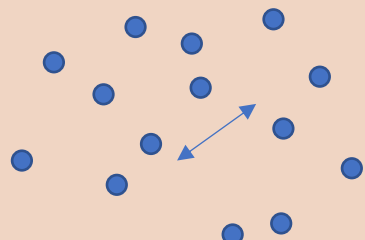
BASIC EXPERIMENTS ON Bose - Einstein Condensates

Giacomo Lamporesi
Pitaevskii BEC Center
CNR-INO / University of Trento
Italy



Space domain

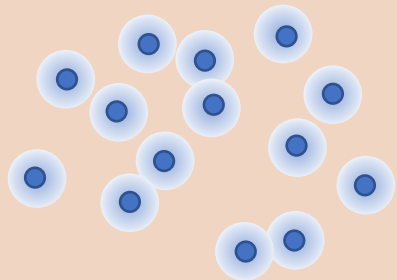
Energy domain



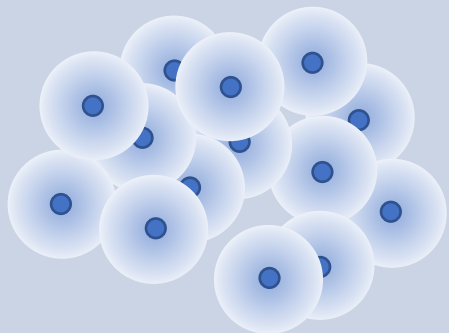
d – interparticle distance

λ – de Broglie wavelength

$$\text{PSD} = (\lambda/d)^3 = n / T^{3/2}$$



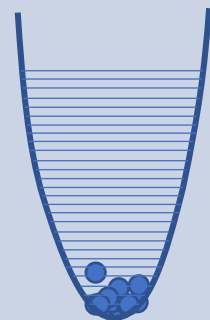
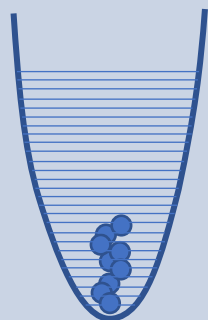
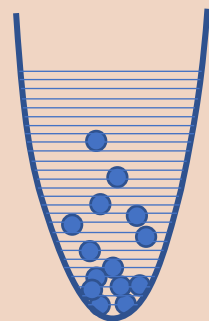
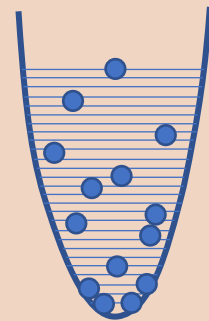
THERMAL GAS



DEGENERATE QUANTUM GAS

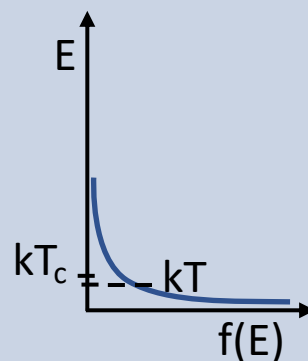
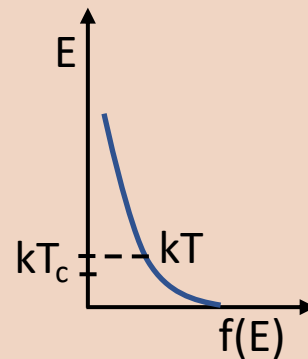
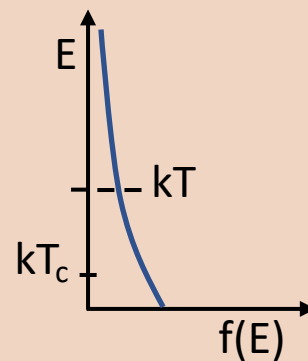
T

T_c



Fermions

Bosons



Condensation and superfluidity

Macroscopic occupation of the ground state

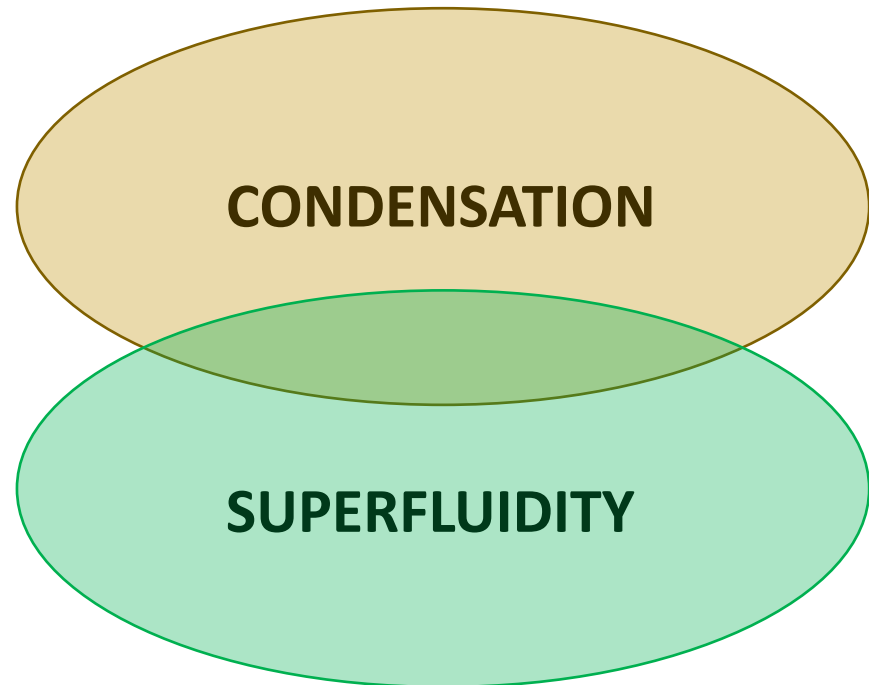
Peaked momentum distribution

Long-range coherence

Absence of viscosity

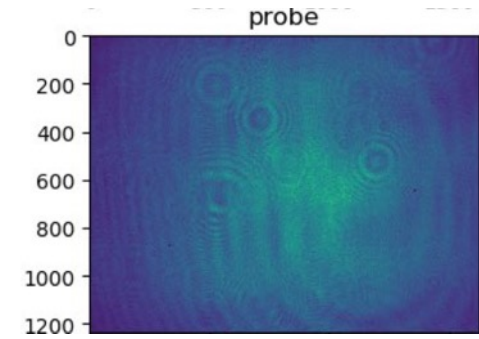
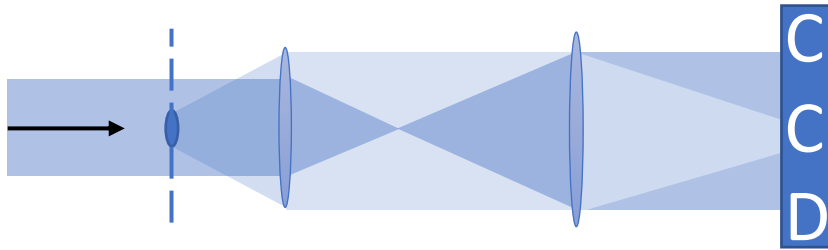
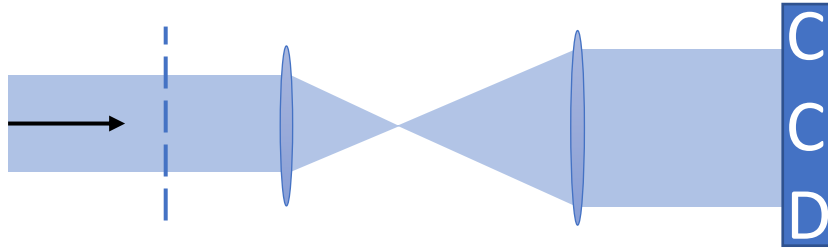
Existence of quantized vortices

Ideal gases

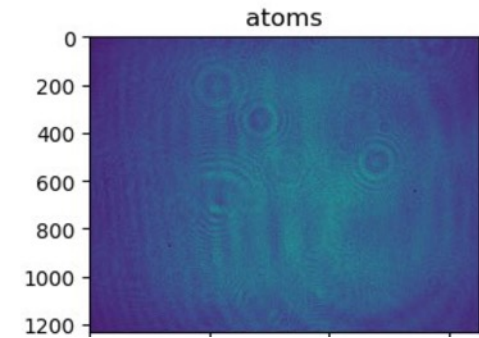


Interacting
Gases in low Dim

Detection - Imaging



$I_0(y, z)$

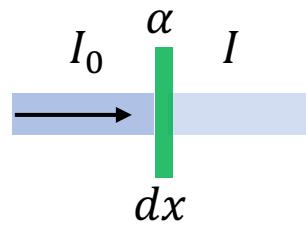


$I(y, z)$

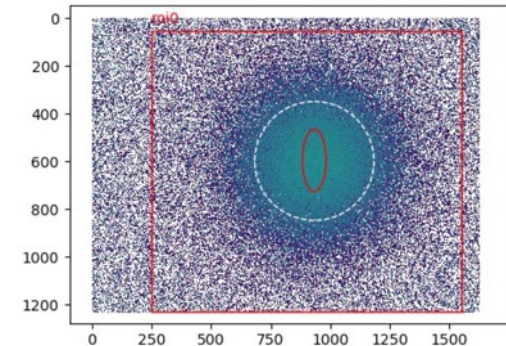
Absorption imaging

Lambert-Beer law

$$dI = -\alpha I dx$$



$$\ln\left(\frac{I_0}{I}\right) = \int \alpha(x) dx = \sigma_0 \int n(x, y, z) dx = \sigma_0 n(y, z)$$



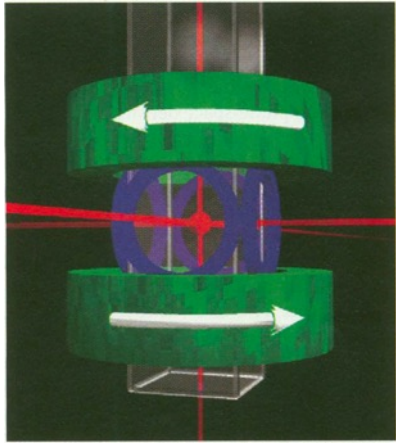
$n(y, z)$



$n(y)$

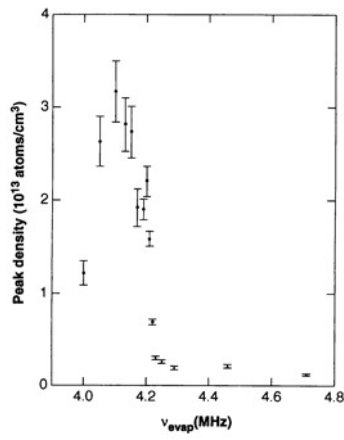
(→ Manuele's talk)

Cornell-Wieman
(JILA, Boulder, Colorado)

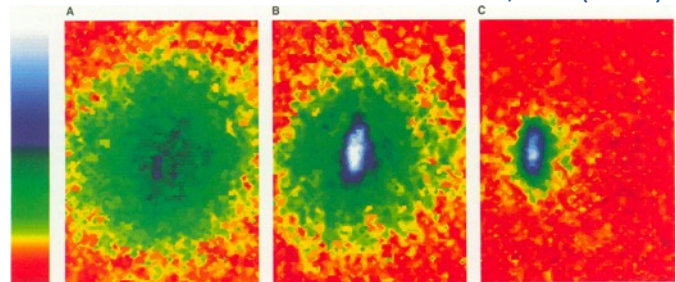


TOP trap
(Quadrupole + rotating bias)

⁸⁷Rb

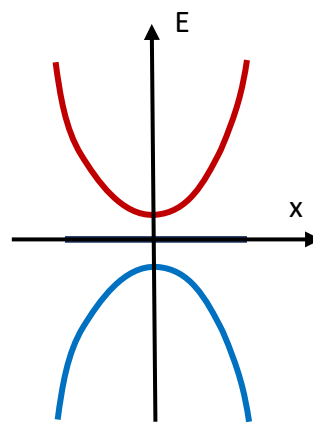
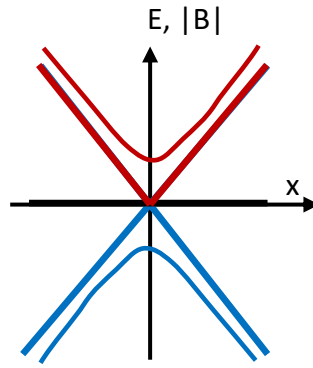


Science 269, 198 (1995)



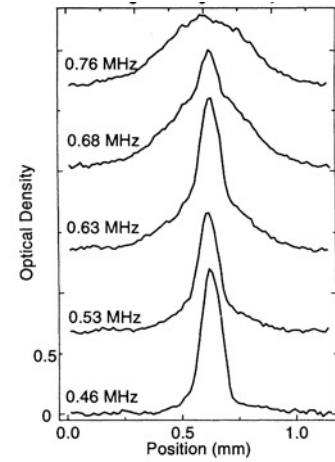
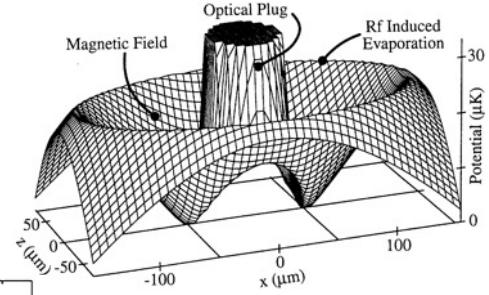
Bimodal distribution after free expansion

Proof of BEC



Ketterle
(MIT, Boston, Massachusetts)

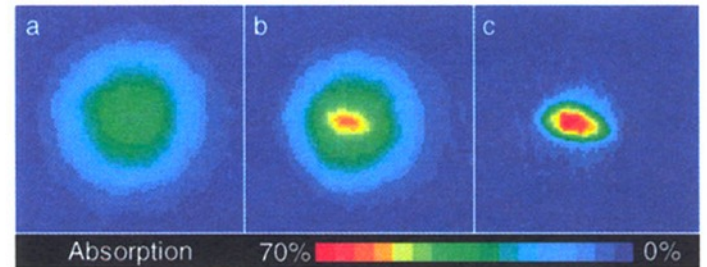
²³Na



Optically plugged magnetic trap

(Quadrupole + repulsive laser)

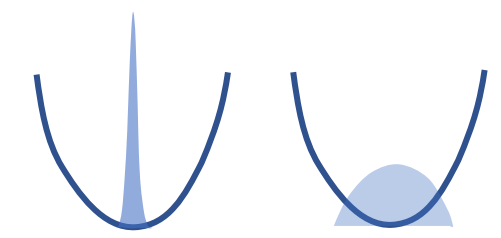
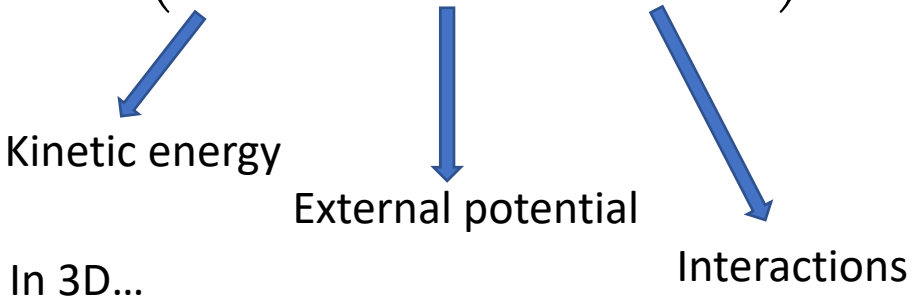
PRL 76, 3969 (1995)



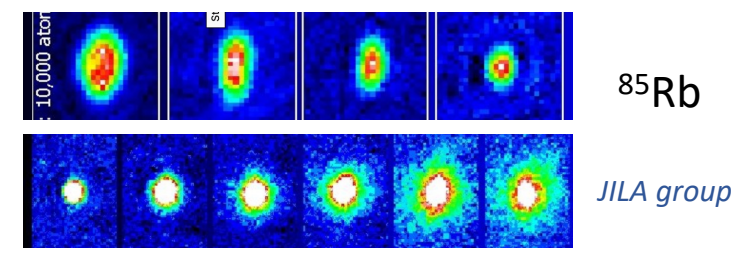
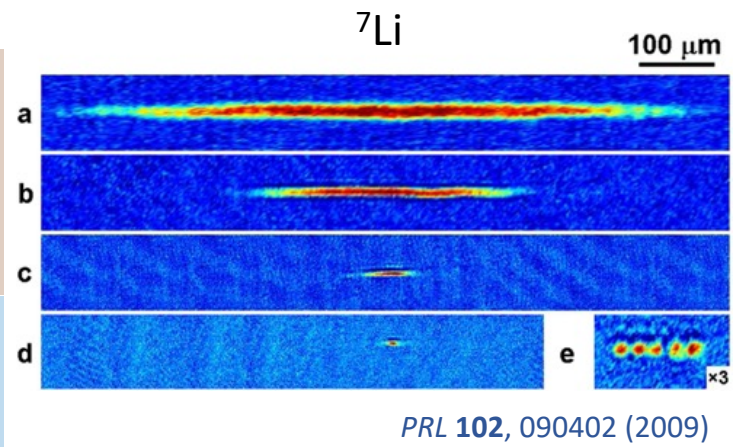
Gross-Pitaevskii equation (\rightarrow Russell's talk)

Tunable interactions (\rightarrow Alessandro's talk)

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + g|\psi(\mathbf{r}, t)|^2 \right) \psi(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t)$$

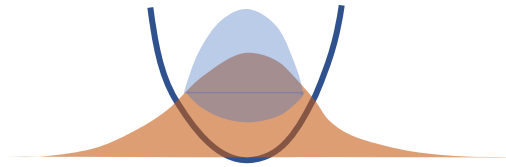


$g > 0$	Repulsive interactions (stable)	BEC of weakly-interacting particles
$g = 0$	No interactions	Ideal BEC
$g < 0$	Attractive interactions (collapse)	«Bose nova»



Free expansion

Elongated harmonic trap



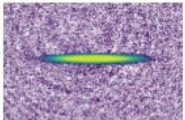
Short expansion:

-high optical density

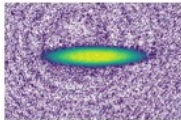
-small features

-hard to distinguish the two components

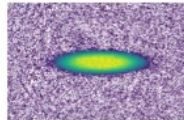
1 ms



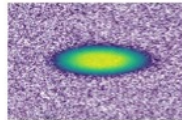
2 ms



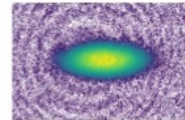
3 ms



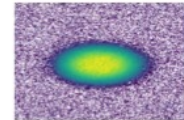
4 ms



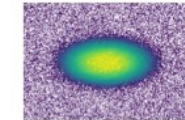
5 ms



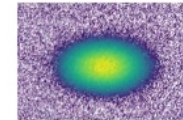
6 ms



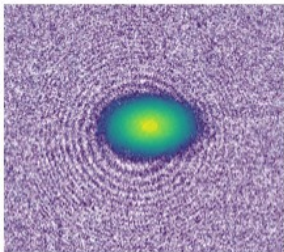
7 ms



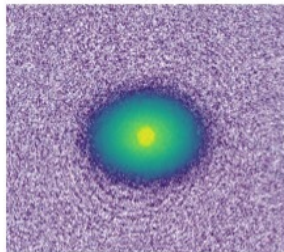
8 ms



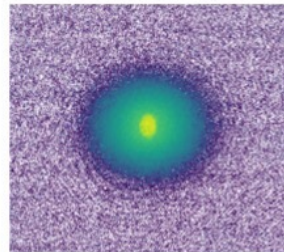
10 ms



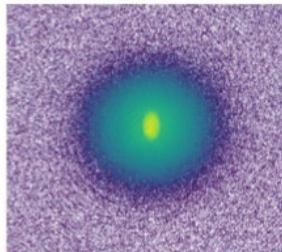
15 ms



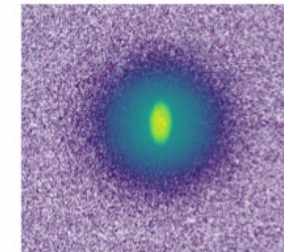
20 ms



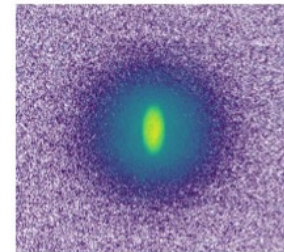
25 ms



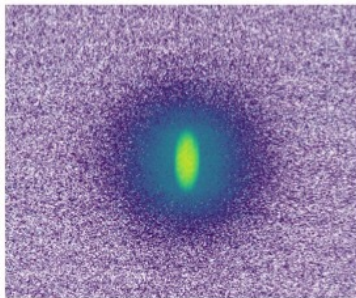
30 ms



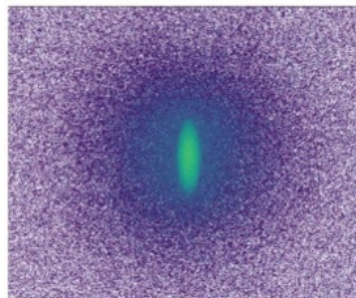
35 ms



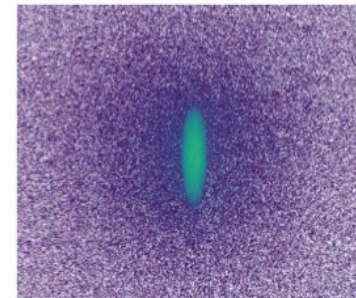
40 ms



60 ms



80 ms



^{23}Na

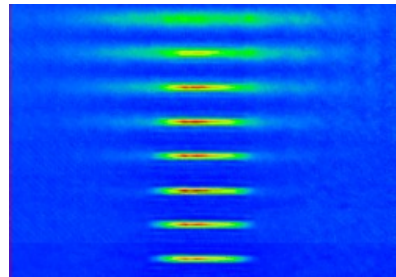
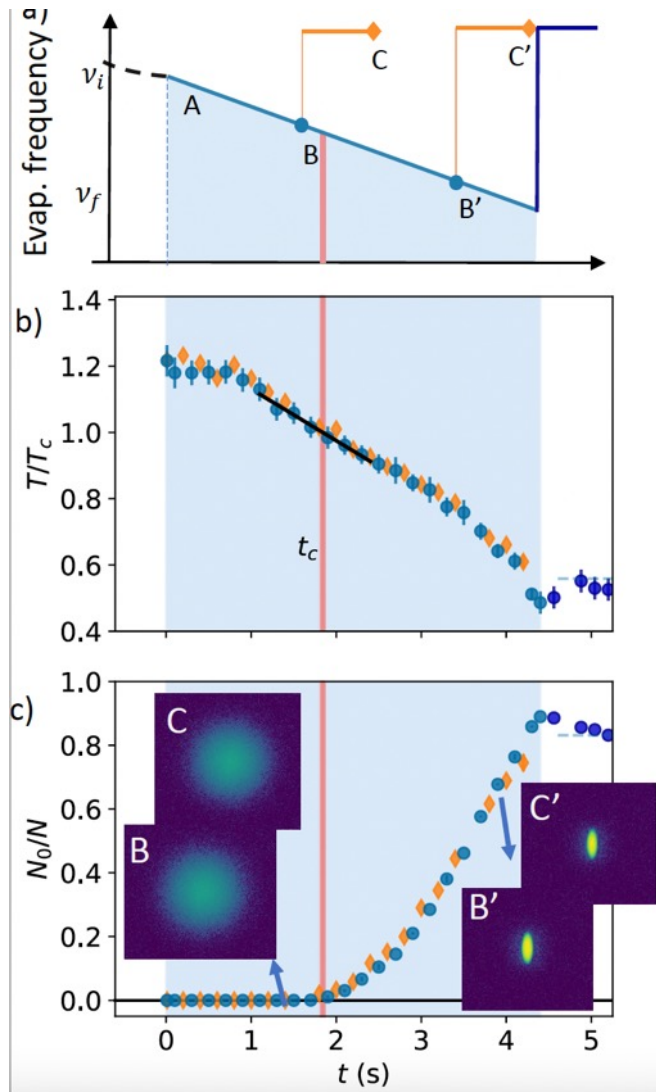
TRENTO group

Long expansion:

-Thermal component tends to $n(k)$ (temperature measurement)

-BEC expansion is dominated by interactions (Aspect ratio inversion)

Cooling across the transition



Science **273**, 84 (1996)

BEC wave function

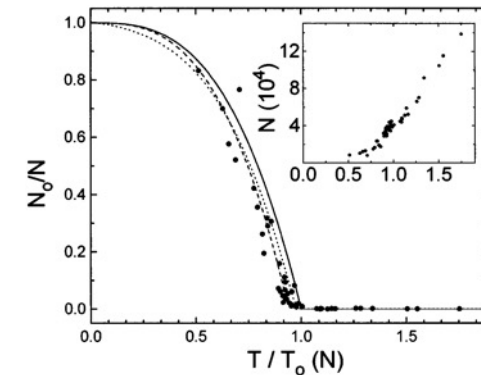
$$\Psi(\mathbf{r}) = \sqrt{n(\mathbf{r})} e^{-i\varphi(\mathbf{r})}$$

Symmetry breaking

$$kT_c = 0.94\hbar\omega N^{1/3}$$

Growth of the order parameter

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c}\right)^3$$



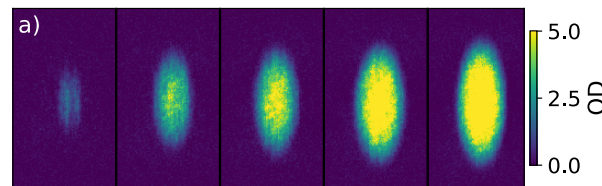
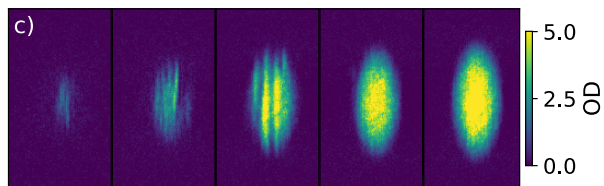
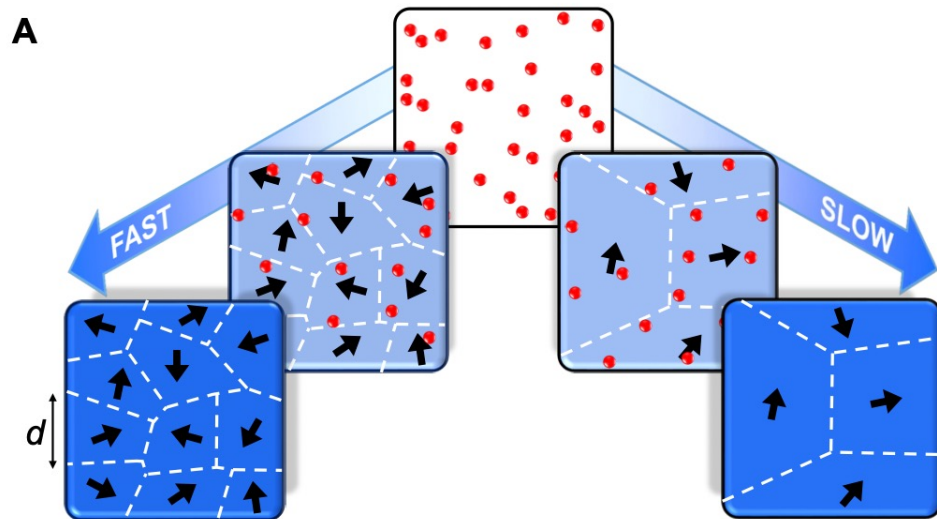
PRL **77**, 4984 (1996)

Condensate formation

$$\Psi(\mathbf{r}) = \sqrt{n(\mathbf{r})} e^{-i\varphi(\mathbf{r})}$$

Symmetry breaking

Local choice of the order parameter



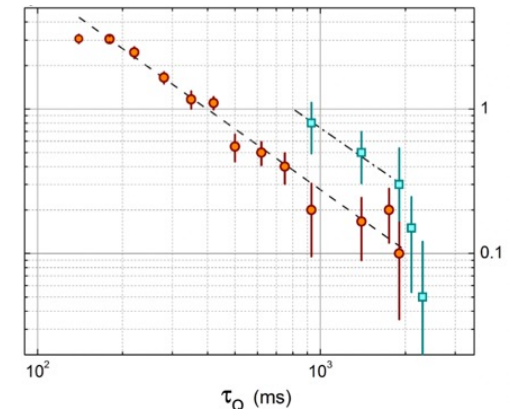
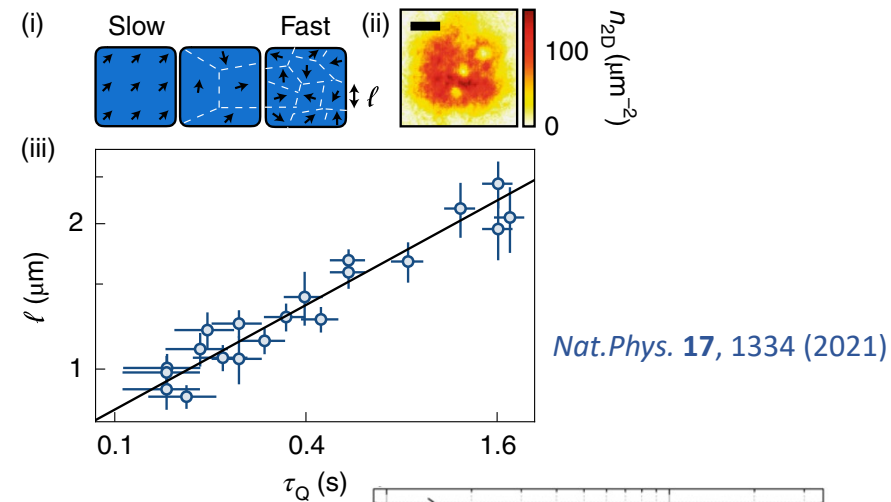
PRA **105**, 033316 (2022)

BEC: second order phase transition
(symmetry breaking)

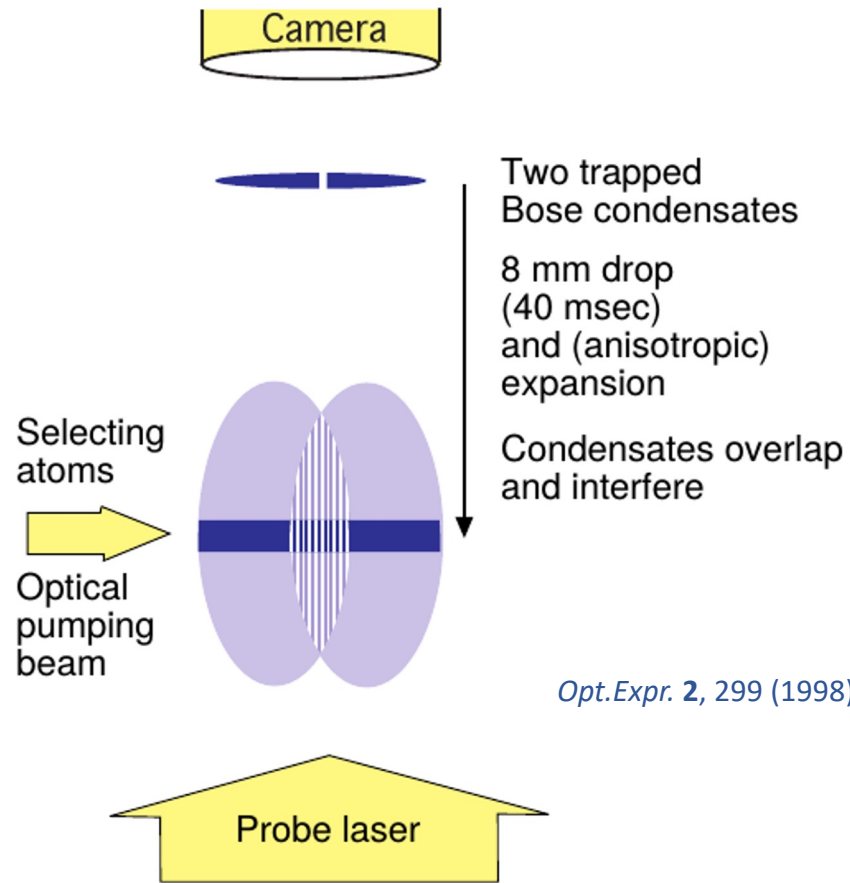
Kibble-Zurek mechanism

Formation of **domains**

Power-law scaling between
domain size and cooling rate



Long-range coherence

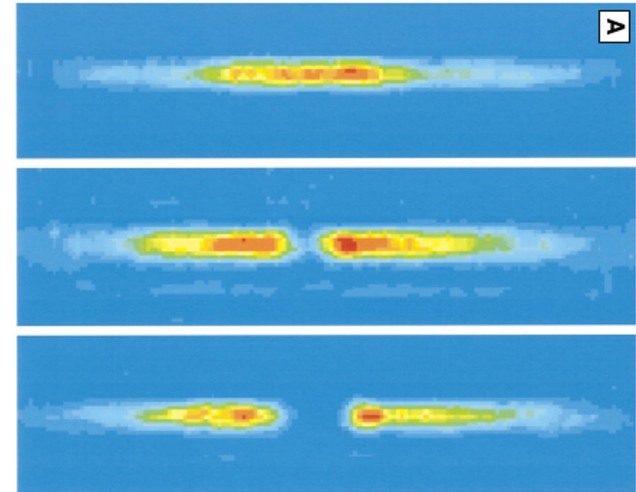


Opt.Expr. 2, 299 (1998)

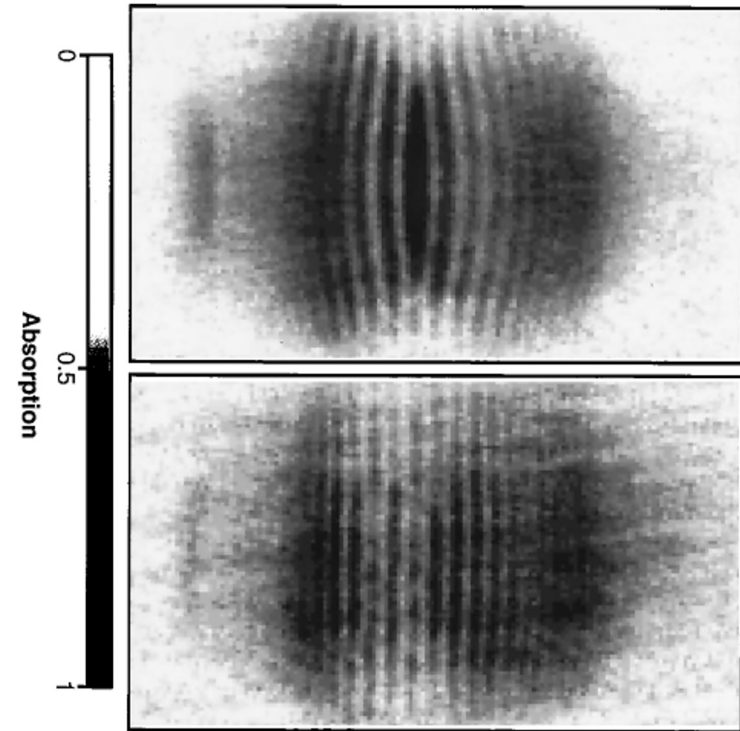
$$p = mv = \frac{h}{\lambda}$$

$$m \frac{d}{t} = \frac{h}{\lambda}$$

$$\lambda = \frac{ht}{md}$$



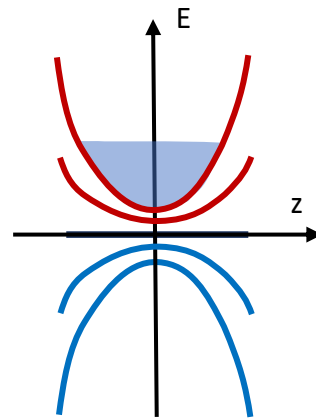
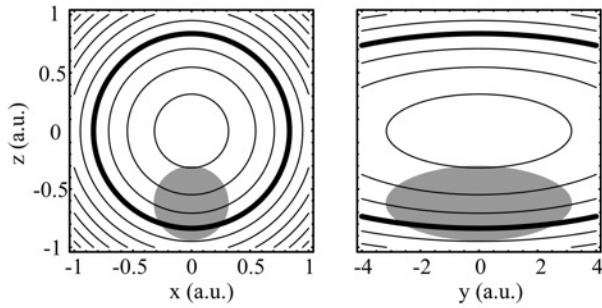
Science 275, 637 (1997)



**Proof
of
BEC**

Long-range coherence

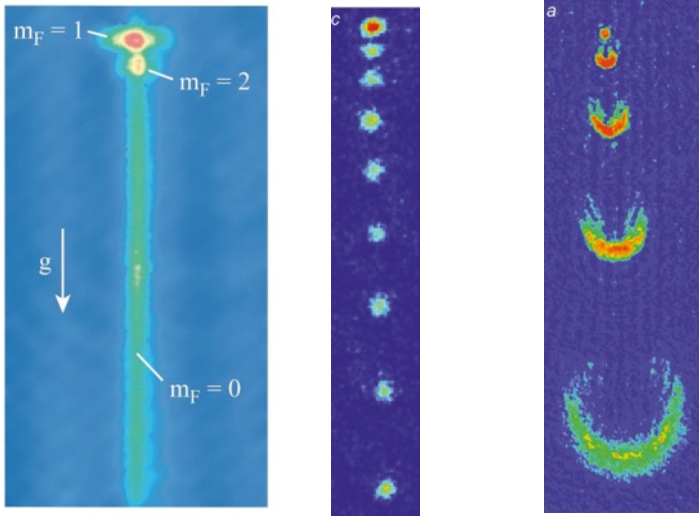
Magnetic trap



Atom laser

**Proof
of
BEC**

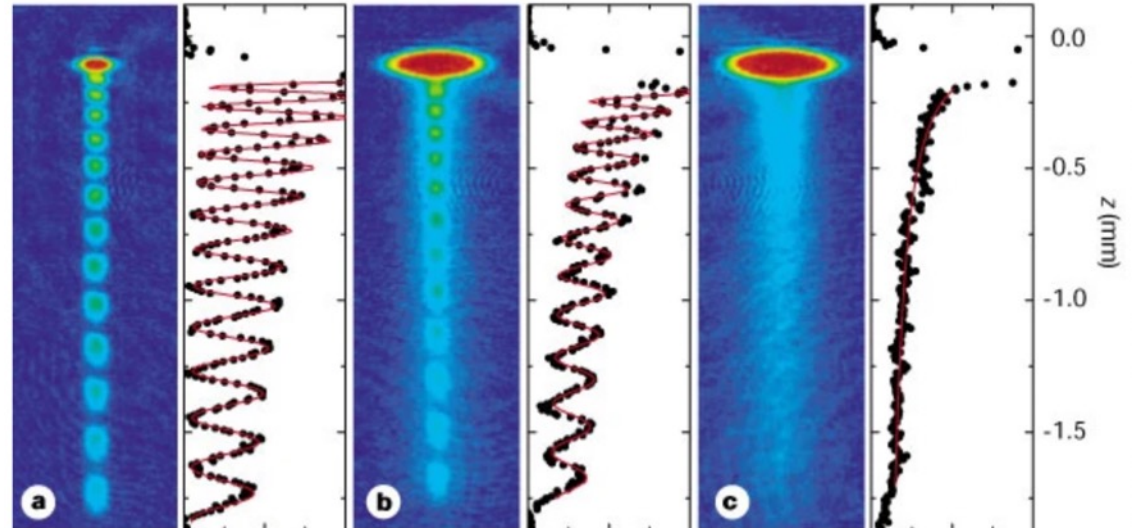
Interference between two CW coherent sources



CW

pulsed

Interaction
effects



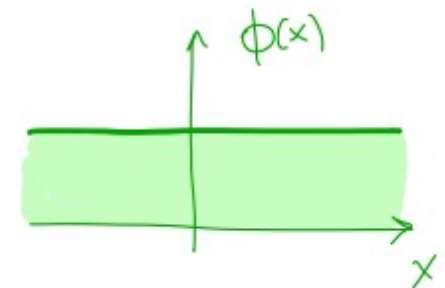
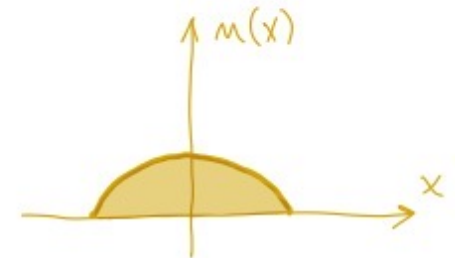
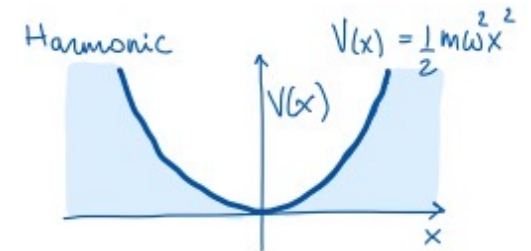
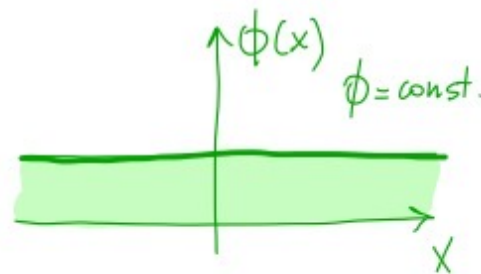
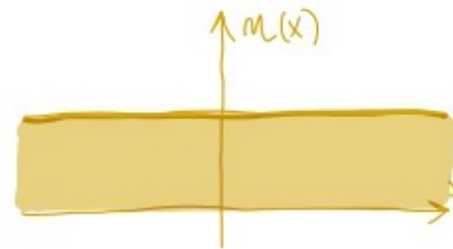
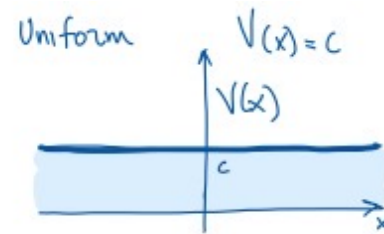
Temperature effects on laser coherence

Gross-Pitaevskii equation

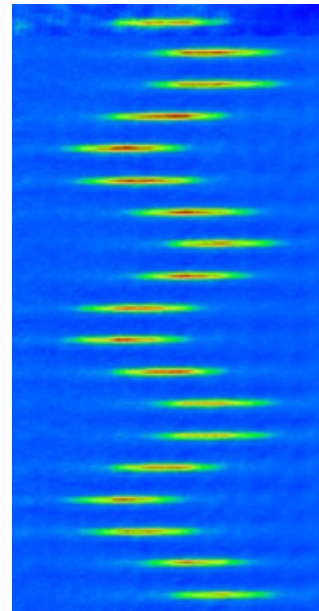
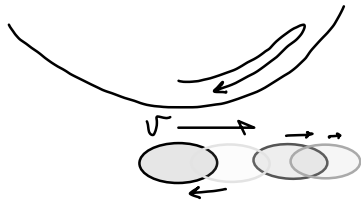
$$\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) + g|\psi(\mathbf{r}, t)|^2 \right) \psi(\mathbf{r}, t) = i\hbar \frac{\partial}{\partial t} \psi(\mathbf{r}, t)$$

$$\Psi(\mathbf{r}) = \sqrt{n(\mathbf{r})} e^{-i\varphi(\mathbf{r})}$$

$$\mathbf{v} = \frac{\hbar}{m} \nabla \varphi$$



Dipole mode



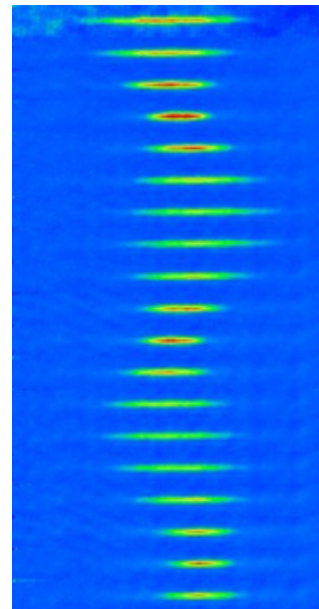
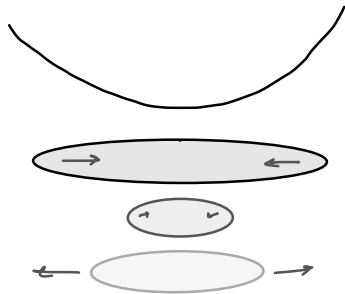
Thermal

ω_x damped

BEC

ω_x undamped
(absence of viscosity)

**Proof
of SF**



Thermal

$2\omega_x$

BEC

$\sqrt{5/2} \omega_x$

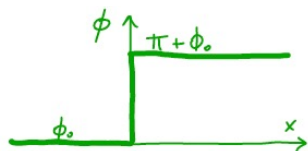
Quadrupole mode

collective modes
(hydrodynamic nature,
reduced compressibility)

Solitons

Localized solitary wave
 Stable solution of the 1D GPE
 Balance between nonlinearity and dispersion

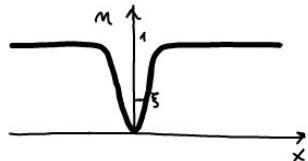
Dark soliton



Phase Profile

$$v = 0$$

$$n(x) = |\psi(x)|^2 = n_0 \tanh^2\left(\frac{x}{\sqrt{2}\xi}\right)$$

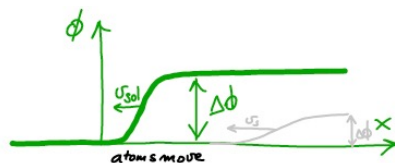


Density Profile

Slow solitons: (deep, narrow, π phase jump)

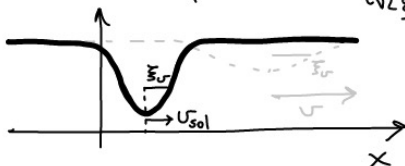
Fast solitons: (small depletion, wide, small $\Delta\phi$)

Grey soliton

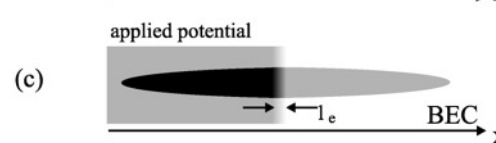
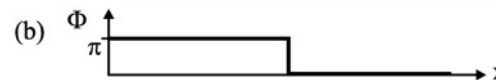


$$v \neq 0$$

$$n_r(x,t) = n_{\min} + (n_0 - n_{\min}) \tanh^2\left(\frac{x - vt}{\sqrt{2}\xi_r}\right)$$



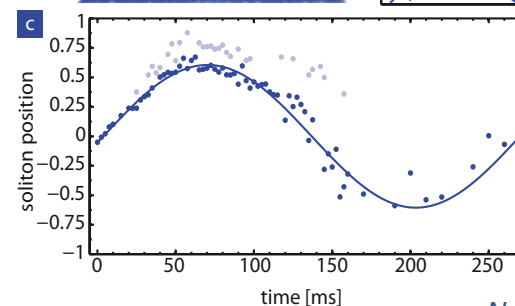
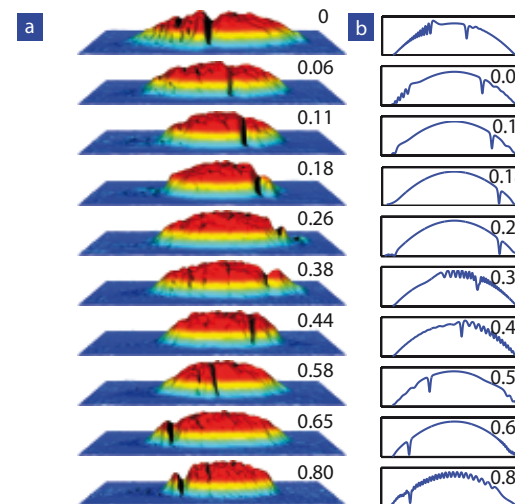
Phase imprinting



$$\Delta\phi = \frac{U \Delta t}{\hbar}$$

PRL **83**, 5198 (1999)

Soliton dynamics



Quantized vortices

Singlevaluedness of the wavefunction \rightarrow phase windings of $n2\pi$ on a closed line

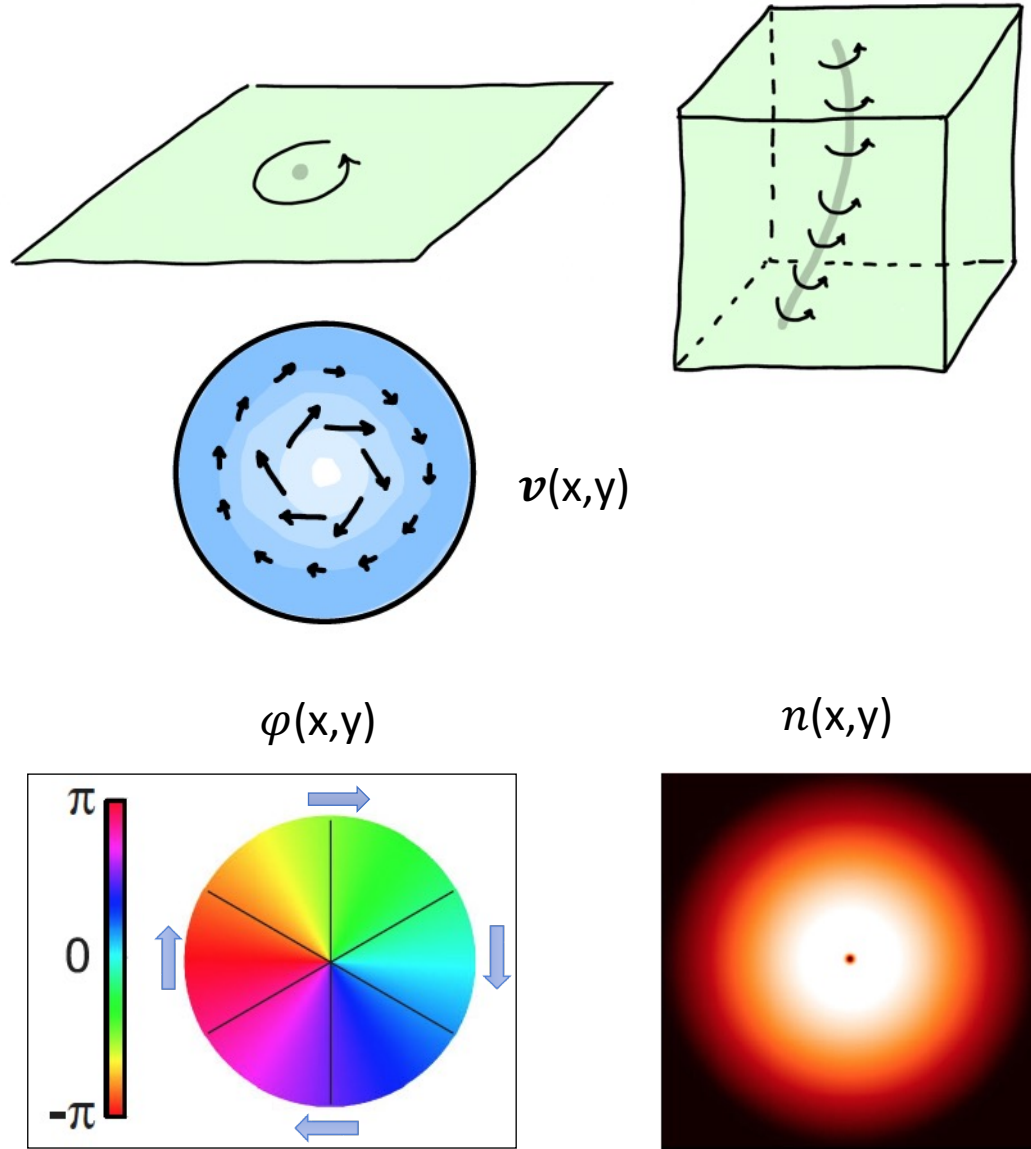
$$\Psi(\mathbf{r}) = \sqrt{n(\mathbf{r})} e^{-i\varphi(\mathbf{r})}$$

$$\mathbf{v} = \frac{\hbar}{m} \nabla\varphi$$

$$v = \frac{l \hbar}{2\pi m r} \rightarrow \text{Density depletion at the vortex core}$$

$$\Gamma = \oint \mathbf{v} \cdot d\mathbf{l} = 2\pi l \frac{\hbar}{m}$$

Quantization of the circulation



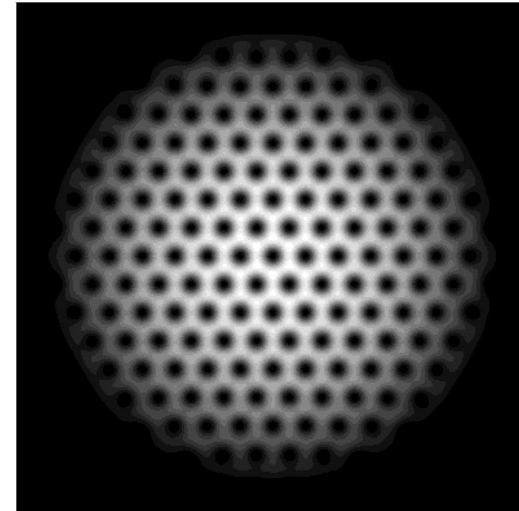
Quantized vortices

Increasing the total angular momentum

Multiply quantized vortices are not stable \rightarrow decay into separated singly quantized vortices

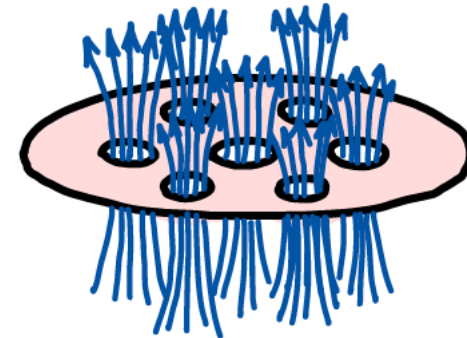
Fast rotation: Abrikosov lattice

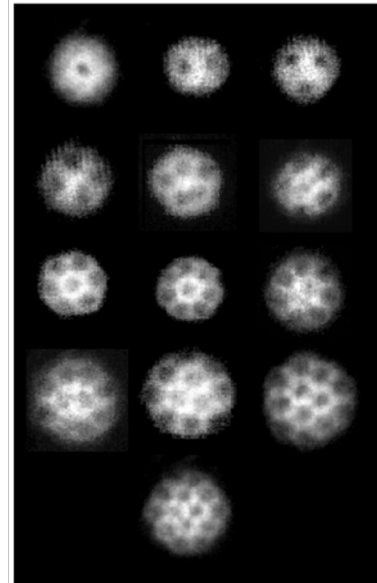
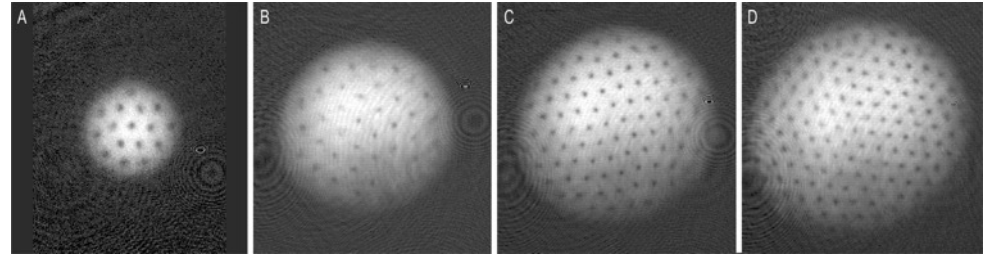
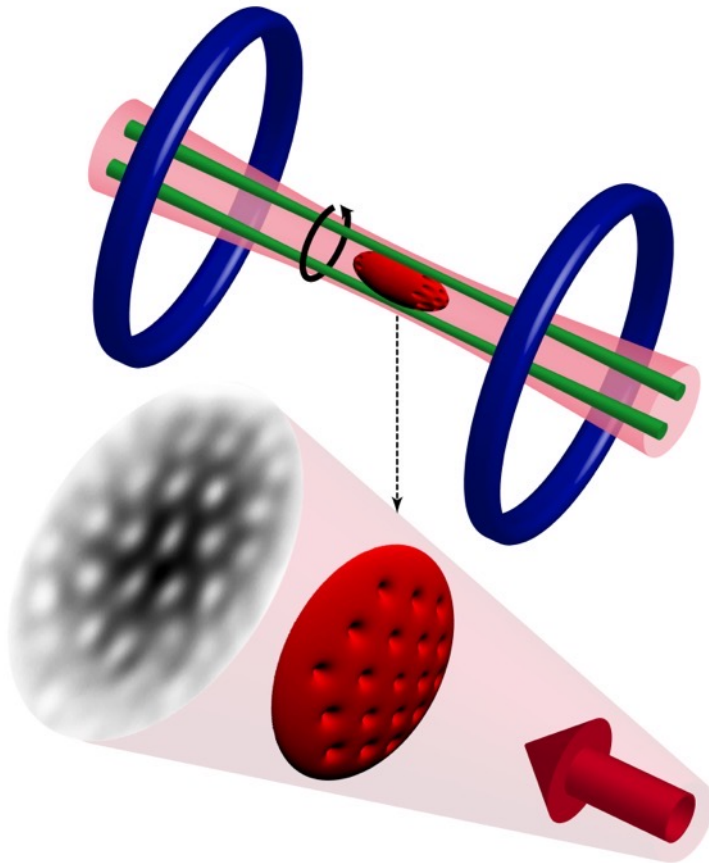
$$n_v = 2\Omega m/h$$



Analogy SUPERFLUIDS - SUPERCONDUCTORS:

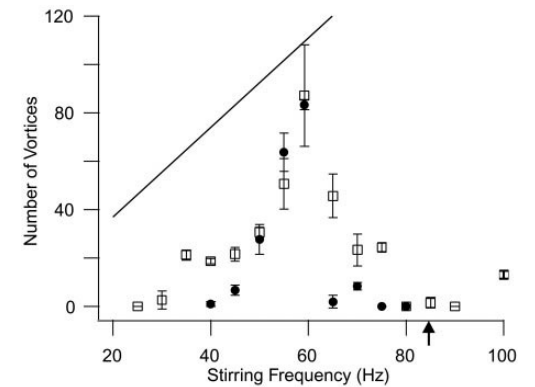
- Neutral atoms + rotation ($\mathbf{L} \cdot \boldsymbol{\Omega}$)
- Charged particles + magnetic field ($\mathbf{S} \cdot \mathbf{B}$)



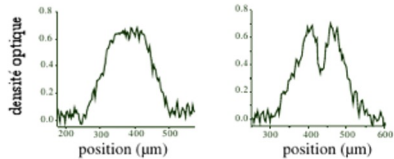
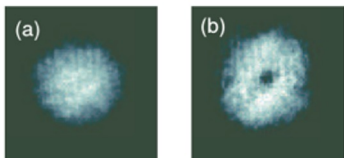


Proof of SF

Science 292, 476 (2001)

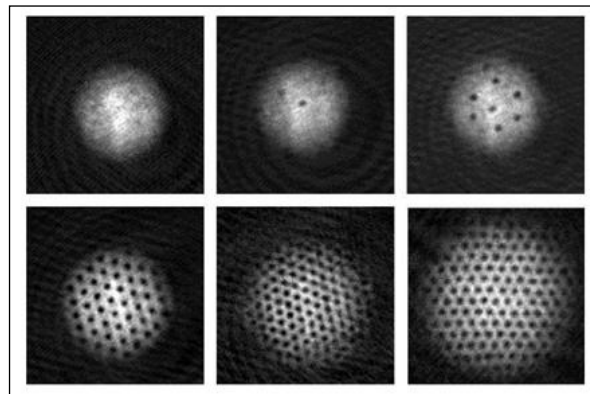


PRL 84, 806 (2000)



JILA group

PRL 84, 806 (2000)



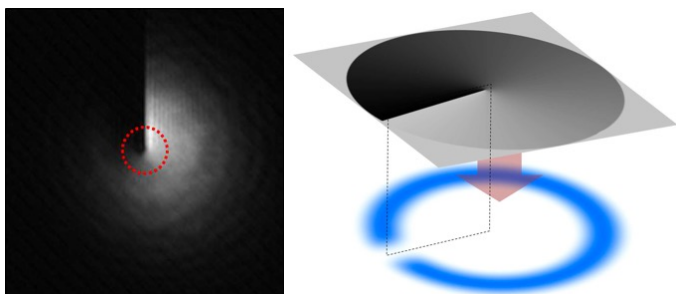
Deform symmetric potential
Rotating the deformation
excites surface modes that let
vortices enter from the outside

Resonance $\omega / \sqrt{2}$
Max rotation ω

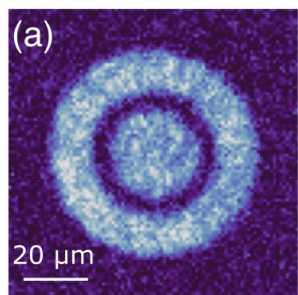
Quantized vortices – IMPRINTING

Phase mask

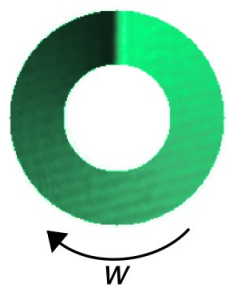
- Critical phase jump
- Low definition at the core



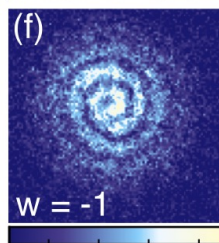
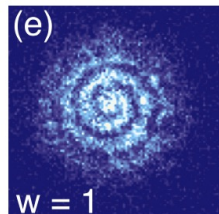
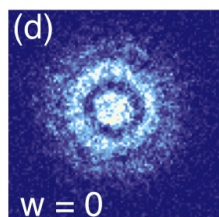
PRA 97, 043615 (2018)



(c) DMD image



$U_0(r, \theta)$ (a.u.)

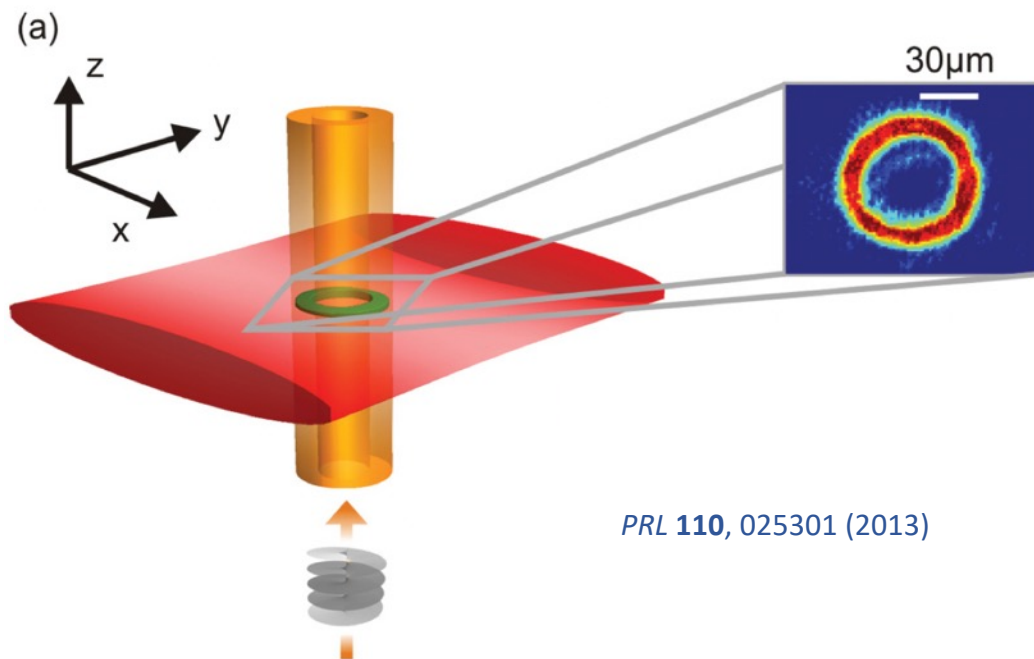


1 2 3 4
 $n(x, y)$ (a.u.)

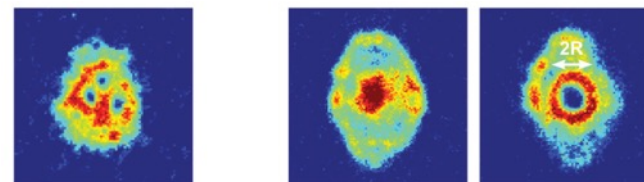
PRX 12, 041037 (2022)

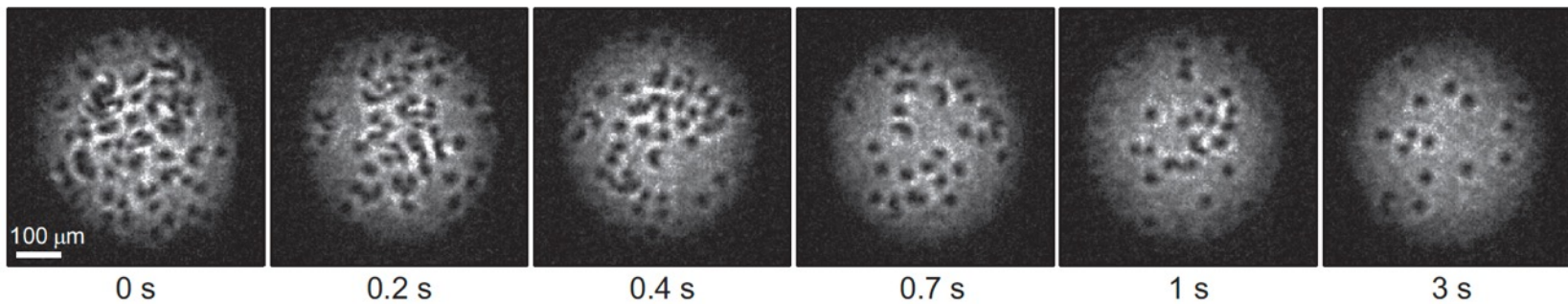
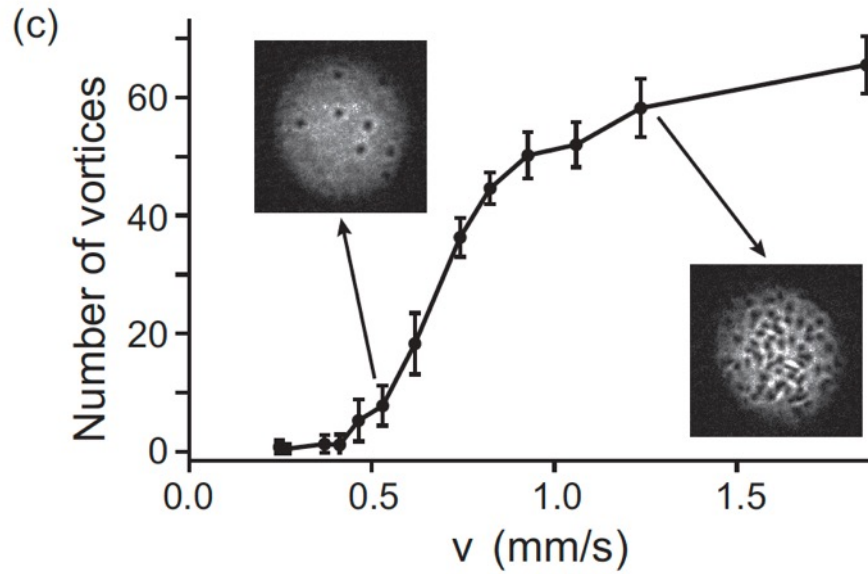
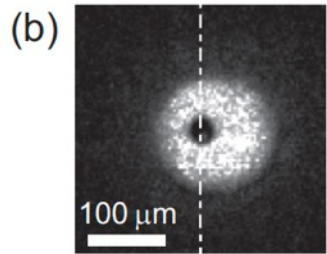
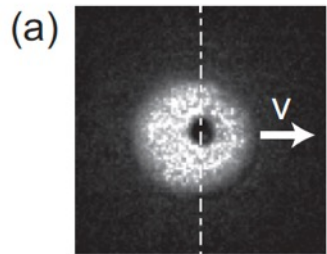
Well definite L

Laguerre-Gauss beam



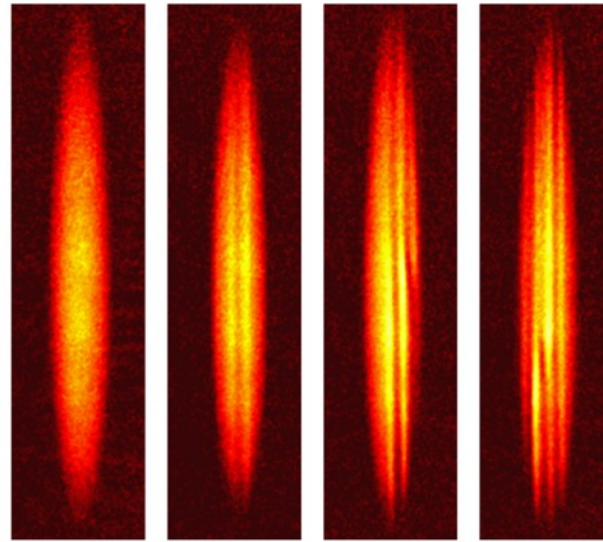
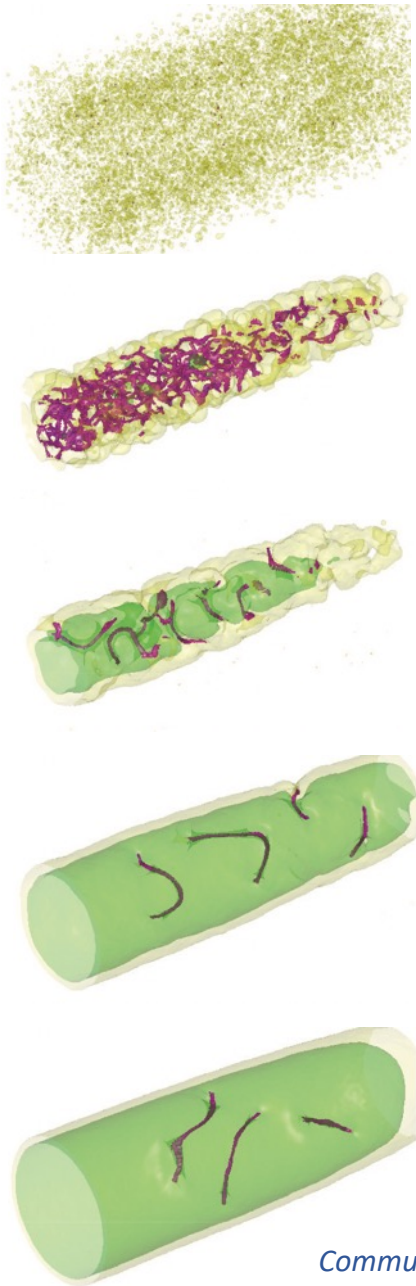
PRL 110, 025301 (2013)



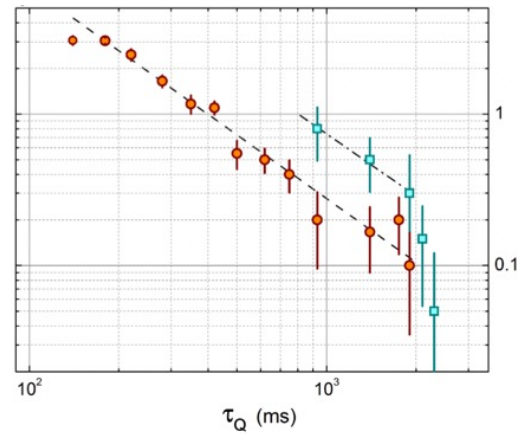


No net angular momentum

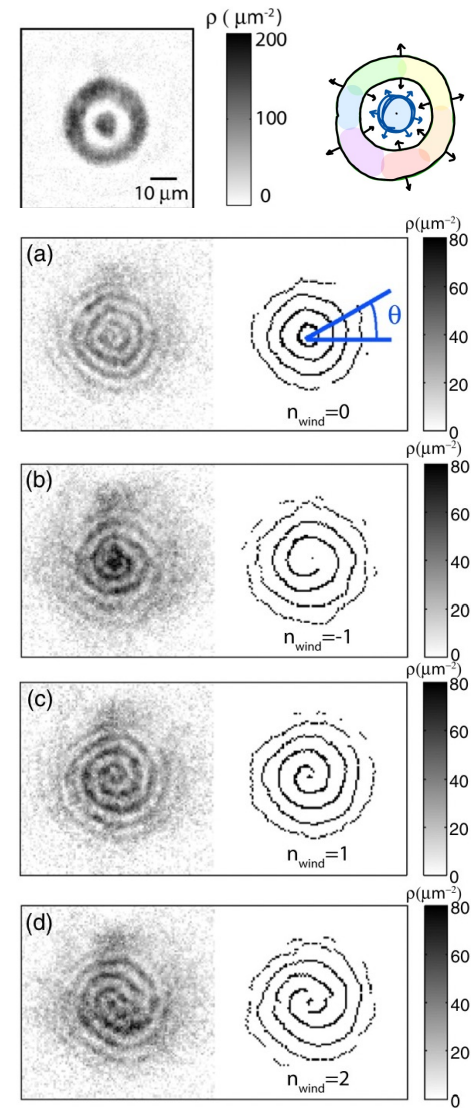
Kibble-Zurek mechanism



Nat.Phys. **9**, 656 (2013)

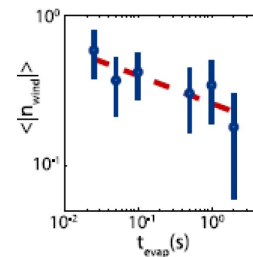


Commun.Phys. **1**, 24 (2018)



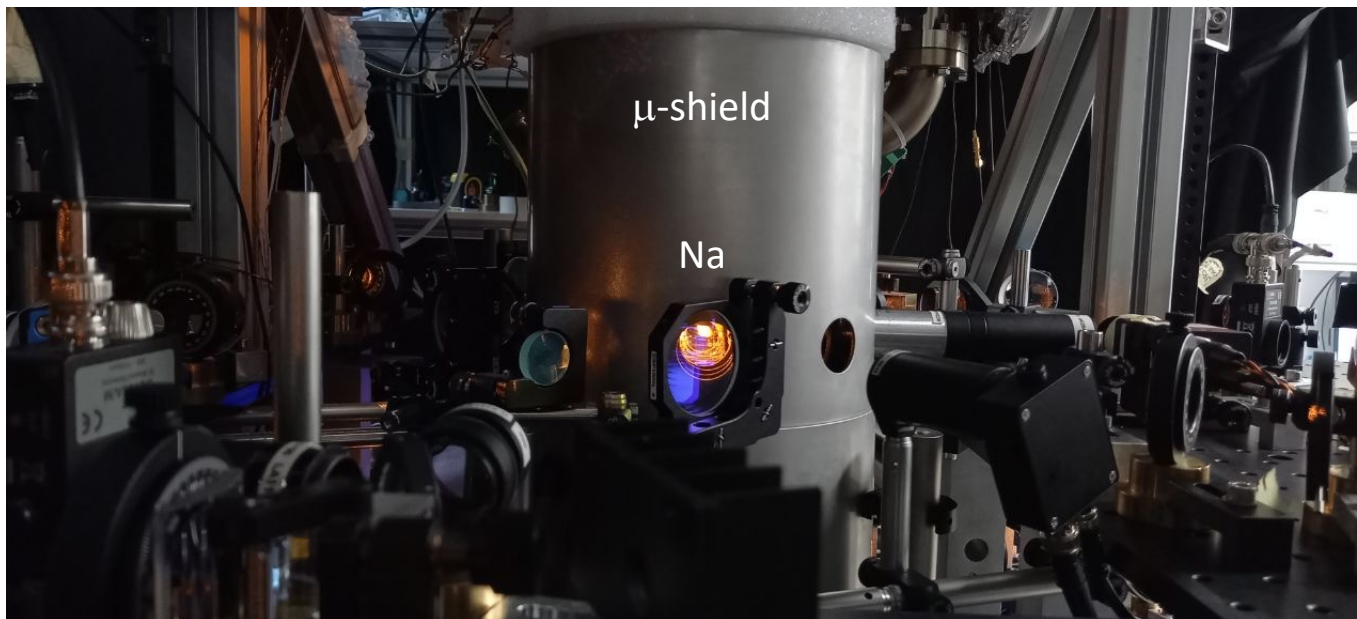
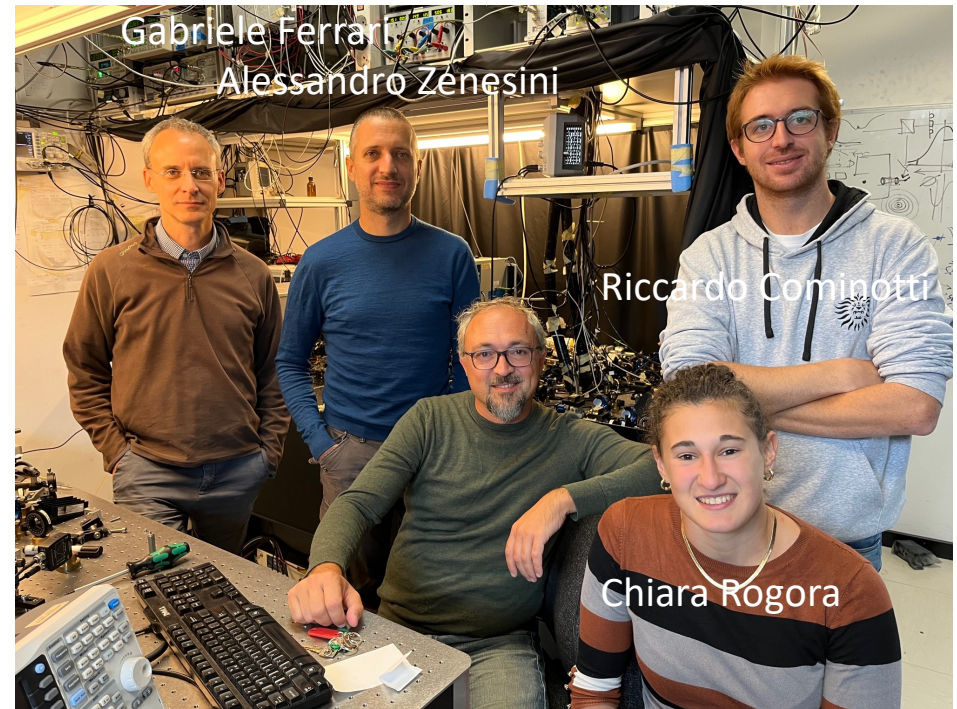
PRL **113**, 135302 (2014)

Defect number



Master and PhD positions available.
If you're interested... let me know!

<https://bec.science.unitn.it>

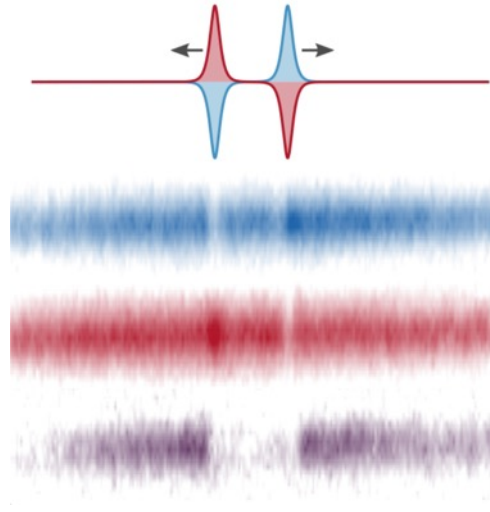


Theory collaborators in Trento:

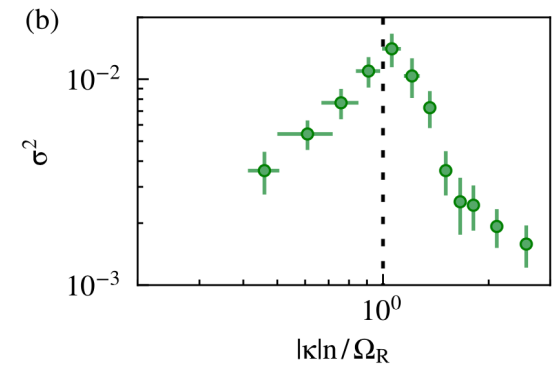
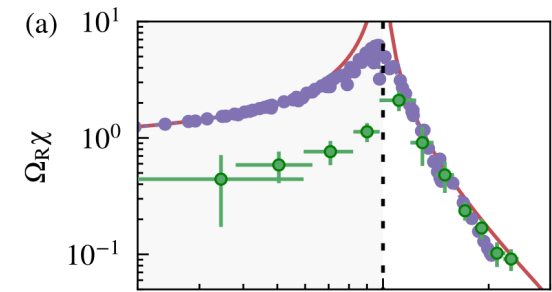
Alessio Recati
Iacopo Carusotto
Anna Berti
Franco Dalfovo
Sandro Stringari

Main Topic: **Spin mixtures**

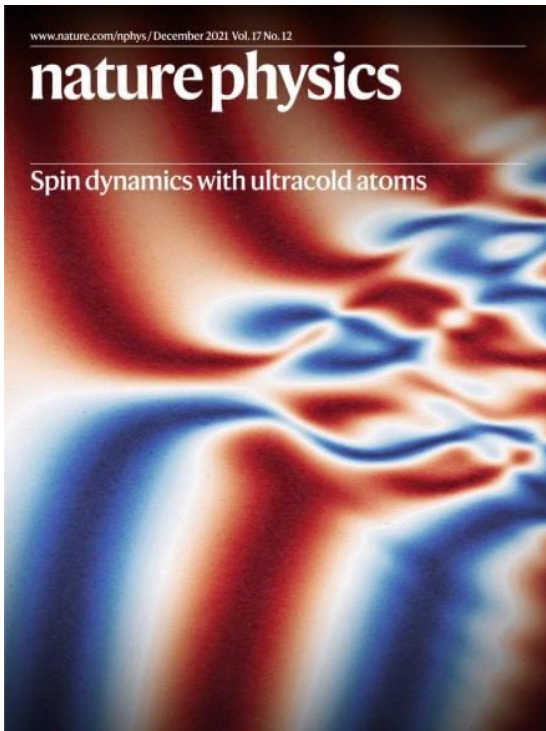
- Observation of magnetic solitons
- Spin dynamics in coherently-coupled mixtures
- Spin Faraday waves
- Simulation of para-ferromagnetic QPT



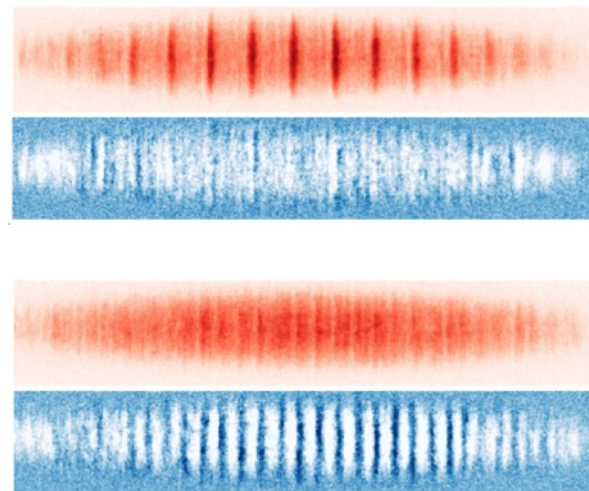
PRL 125, 030401 (2020)



PRX 13, 021037 (2023)



Nat. Phys. 17, 1359 (2021)



PRL 128, 210401 (2022)