Optical lattices (and 1D systems)

Introductory Course on Ultracold Quantum Gases 2023

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quantummatter.at

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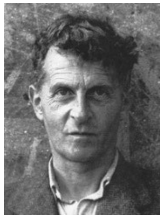
2019-2023



Wittgenstein

Structure of the lecture

Basic introduction to Hubbard models

Interactions and tunneling in the BHM

Basic introduction into 1D systems

Strongly interacting bosons in 1D systems

Recommended reading

Many-body physics with ultracold gases,
I. Bloch, J. Dalibard, and W. Zwerger, Rev. Mod. Phys. 80, 885 (2008)

Engineering novel optical lattices,
P. Windpassinger and K. Sengstock, Rep. Prog. Phys. 76 086401 (2013)

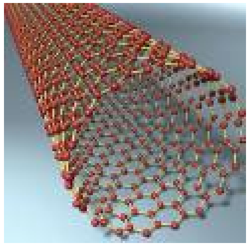
Non-standard Hubbard models in optical lattices: a review,
O. Dutta et al., Rep. Prog. Phys. 78, 066001 (2015)

Quantum simulations with ultracold atoms in optical lattices,
C. Gross and I. Bloch, Science 357, 995 (2017)

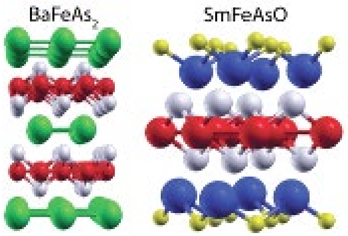
Motivation

Investigating Bose/Fermi-Hubbard models

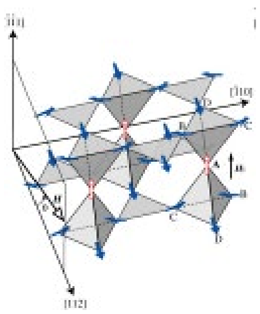
Carbon nanotubes (1D)



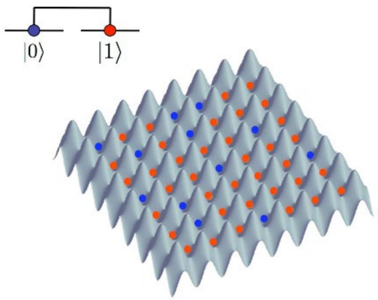
High-temperature superconductors (2D)



Spin-Ice magnetism (3D)

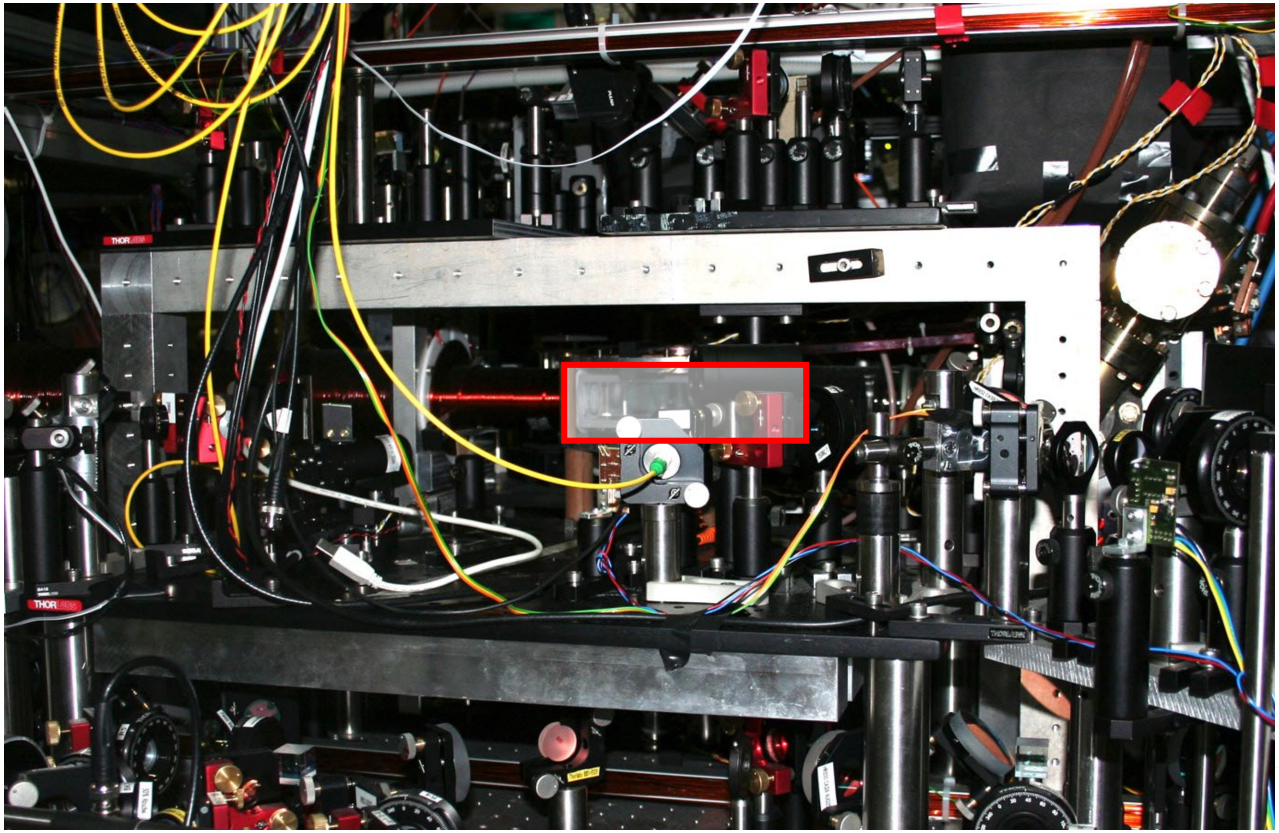


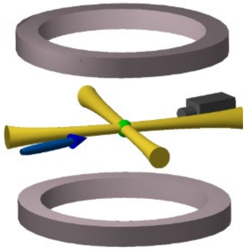
Quantum computation (2D)

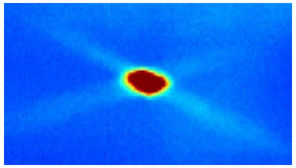


Basis for all lattice experiments: cold atoms

An apparatus like this:

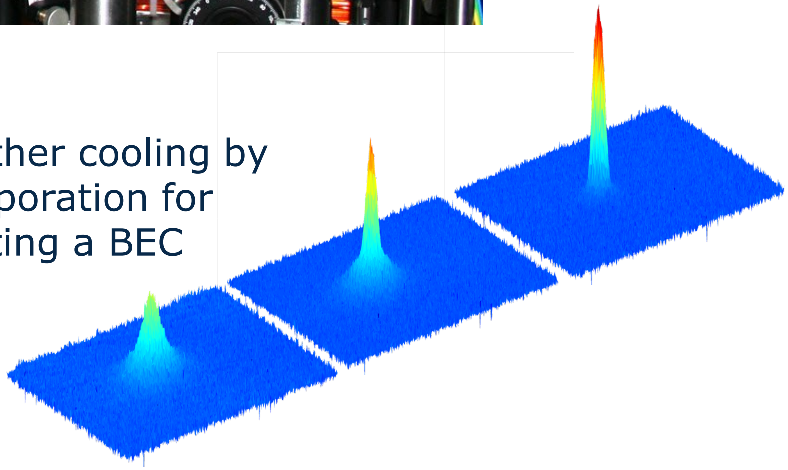






Trapping and cooling atoms down to µK

Further cooling by evaporation for getting a BEC

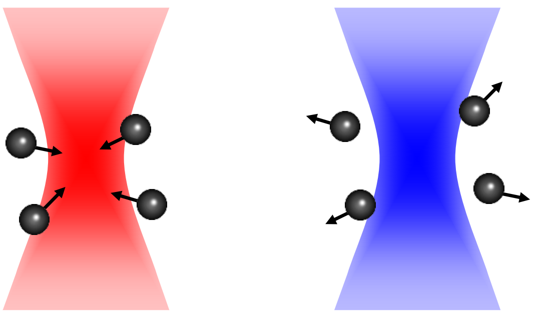


Basic concepts of lattice physics

Creation of an optical lattice

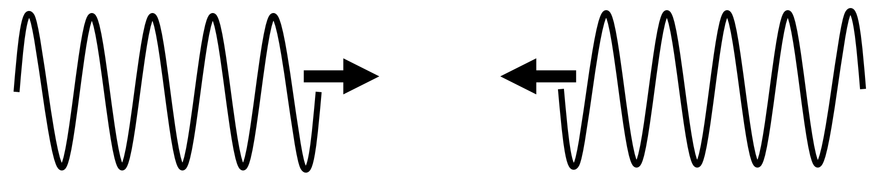
Dipole traps

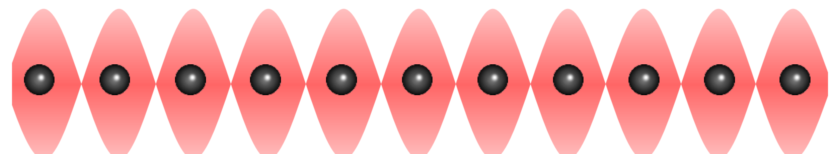
Focused laser light creates attractive or repulsive forces depending on the frequency detuning to the atomic transition



red-detuned **blue-detuned**

Interference

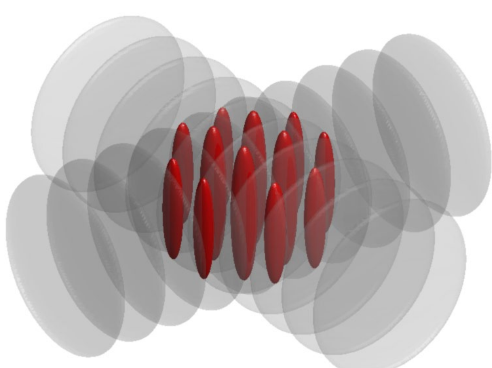




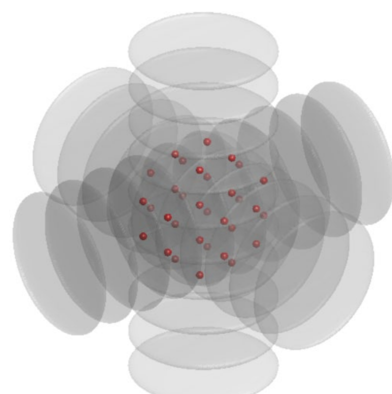
Different lattice configurations

1D lattice

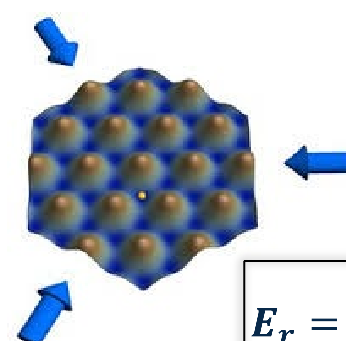
2D lattice



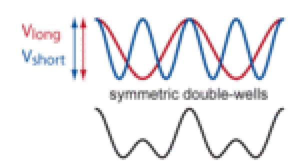
3D lattice



Hexagonal lattice



Superlattice



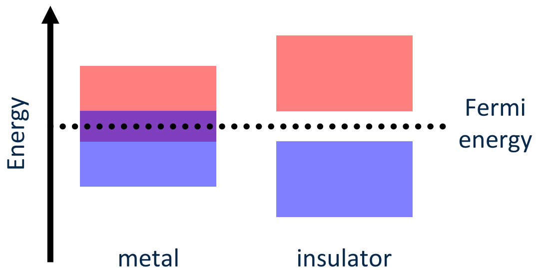
$$E_r = \frac{\hbar^2 k^2}{2m}$$ **recoil energy**

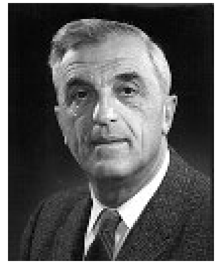
Basic concepts of lattice physics

Original concept

F. Bloch

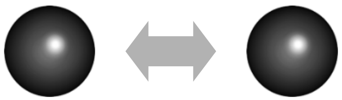
Energy bands + energy gaps





N. F. Mott

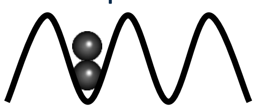
Interactions



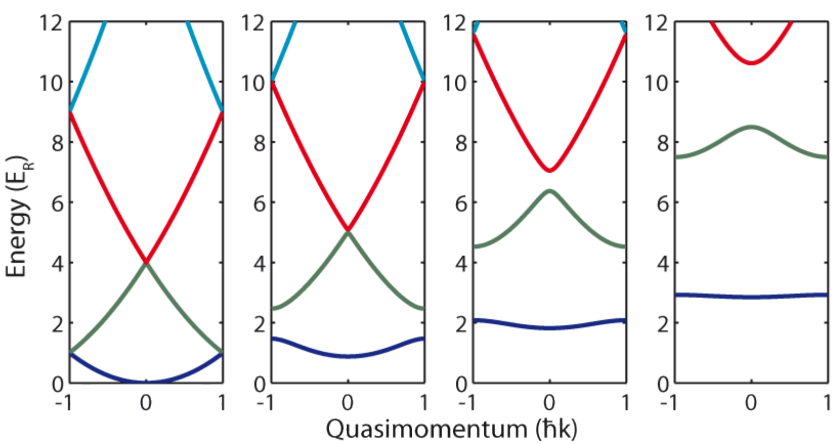
Additional energy gap

J. Hubbard

Local particles

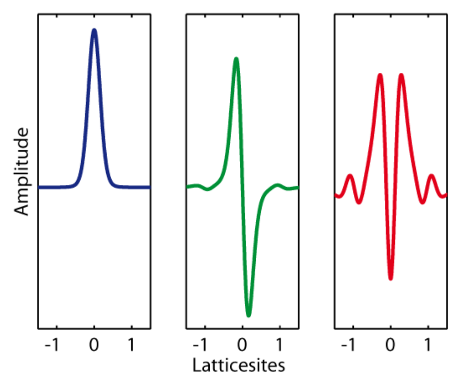


Band structure optical lattice



Localized wavefunctions optical lattice:

Wannier functions $w_i(x)$



Controllable interactions

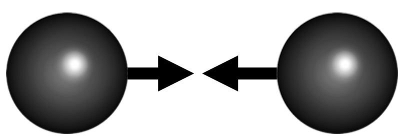
Interactions

Classical example

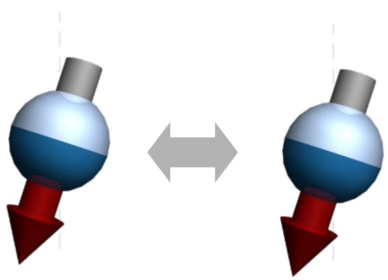
attractive repulsive

Ultracold gases

Short-range interactions, e.g.
Contact interaction

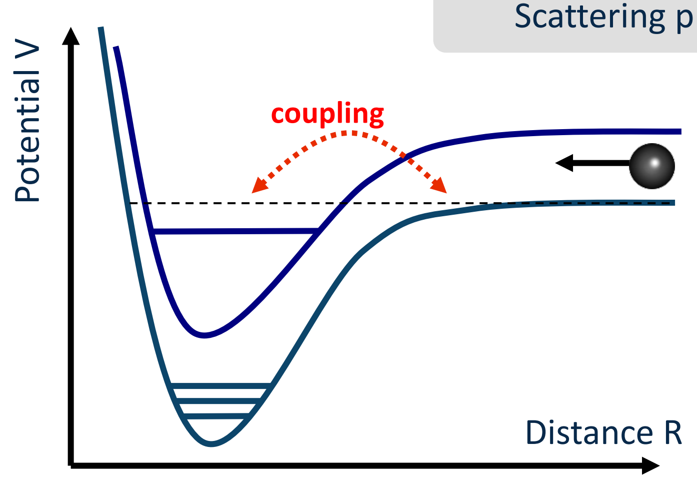


Long-range interactions, e.g.
dipole-dipole

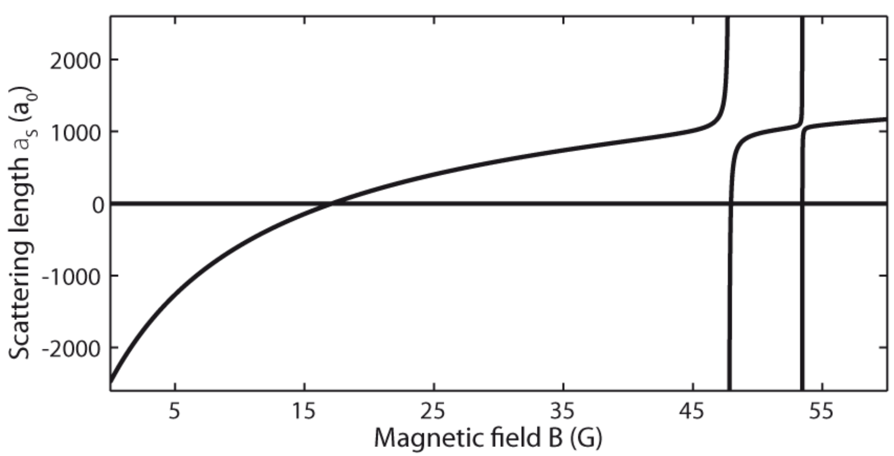


Contact interactions

Scattering process



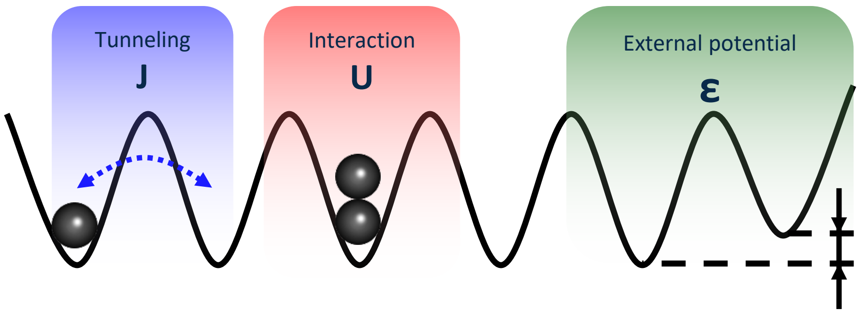
Example: Scattering length in Cs |F=3, m_F=3>



Basic concepts of lattice physics

The standard Bose-Hubbard model

$$\hat{H} = -J \sum_{<i,j>} \hat{a}_i^\dagger \hat{a}_j + \sum_i \frac{U}{2} \hat{n}_i (\hat{n}_i - 1) + \sum_i \epsilon_i \hat{n}_i$$



Tunneling matrix element

$$J = -\int dx w_0(x) \left(-\frac{\hbar^2}{2m} \nabla^2 + V_0 \sin^2(kx) \right) w_0(x-d)$$

On-site interaction energy

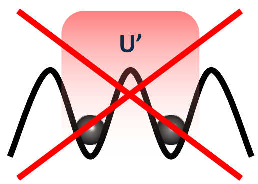
$$U = \int d\mathbf{x} d\mathbf{x}' w_0(\mathbf{x})^2 g(\mathbf{x},\mathbf{x}') w_0(\mathbf{x}')^2$$

$$U \propto a$$

External energy shift

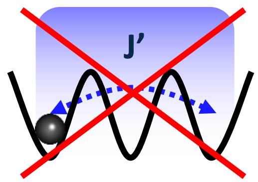
$$\epsilon_i = \int d\mathbf{x} |w_0(\mathbf{x}-\mathbf{x}_i)|^2 V(\mathbf{x}-\mathbf{x}_i)$$

Approximations



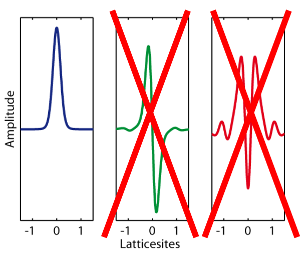
Interactions

No nearest-neighbor interaction



Tunneling

No next nearest-neighbor tunneling



Bloch bands

Higher Bloch bands omitted

Interaction potential

Simple non-regularized pseudopotential

$$g(\mathbf{x},\mathbf{x}') = \frac{4\pi\hbar^2 a}{m} \delta(\mathbf{x}-\mathbf{x}') \qquad U = g \int d\mathbf{x} |w_0(\mathbf{x})|^4$$

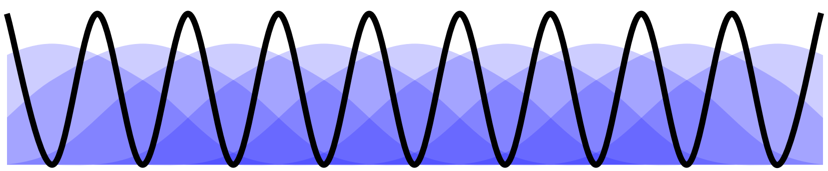
Groundstates and phase diagram

Groundstates at T=0

Superfluid (SF)

J >> U

- Delocalized particles
- Coherent phase
- No excitation gap



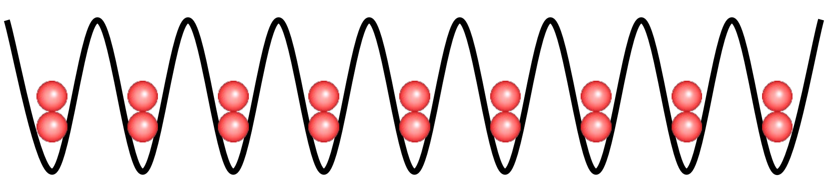
Mott insulator (MI)

J << U

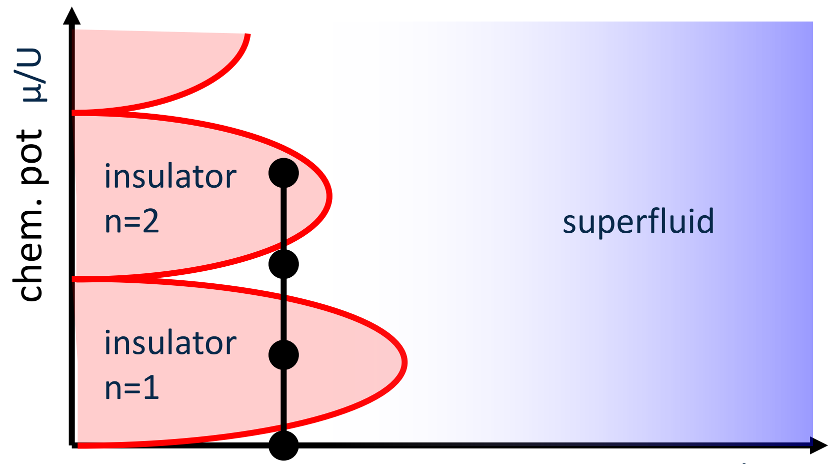
- Localized particles
- No phase coherence
- Excitation gap

critical value

$$\frac{U}{6J} \approx 5$$



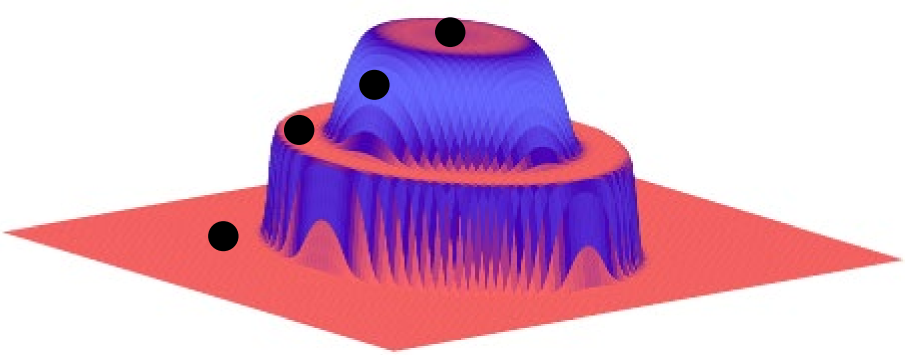
Phase diagram



Experiment

External confinement

‘wedding cake structure’



Observation of the phase transition

Bosons (2002)

articles

Quantum phase transition from a superfluid to a Mott insulator in a gas of ultracold atoms

Markus Greiner*, Olaf Mandel*, Tilman Esslinger†, Theodor W. Hänsch* & Immanuel Bloch*

** Sektion Physik, Ludwig-Maximilians-Universität, Schellingstrasse 4/III, D-80799 Munich, Germany, and Max-Planck-Institut für Quantenoptik, D-85748 Garching, Germany*
† Quantenelektronik, ETH Zürich, 8093 Zurich, Switzerland

For a system at a temperature of absolute zero, all thermal fluctuations are frozen out, while quantum fluctuations prevail. These microscopic quantum fluctuations can induce a macroscopic phase transition in the ground state of a many-body system when the relative strength of two competing energy terms is varied across a critical value. Here we observe such a quantum phase transition in a Bose–Einstein condensate with repulsive interactions, held in a three-dimensional optical lattice potential. As the potential depth of the lattice is increased, a transition is observed from a superfluid to a Mott insulator phase. In the superfluid phase, each

Fermions (2008)

Metallic and Insulating Phases of Repulsively Interacting Fermions in a 3D Optical Lattice

U. Schneider,[1] L. Hackermüller,[1] S. Will,[1] Th. Best,[1] I. Bloch,[1,2]* T. A. Costi,[3]
R. W. Helmes,[4] D. Rasch,[4] A. Rosch[4]

The fermionic Hubbard model plays a fundamental role in the description of strongly correlated materials. We have realized this Hamiltonian in a repulsively interacting spin mixture of ultracold ^{40}K atoms in a three-dimensional (3D) optical lattice. Using in situ imaging and independent control of external confinement and lattice depth, we were able to directly measure the compressibility of the quantum gas in the trap. Together with a comparison to ab initio dynamical mean field theory

LETTERS

A Mott insulator of fermionic atoms in an optical lattice

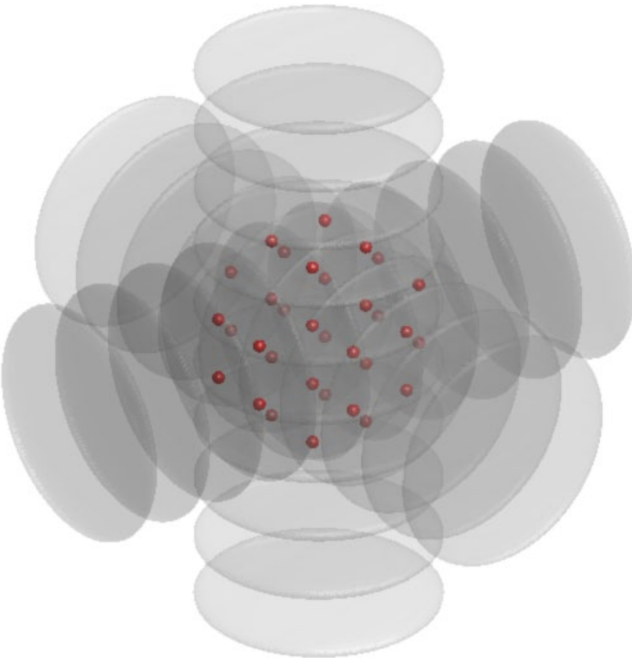
Robert Jördens[1]*, Niels Strohmaier[1]*, Kenneth Günter[1,2], Henning Moritz[1] & Tilman Esslinger[1]

Strong interactions between electrons in a solid material can lead to surprising properties. A prime example is the Mott insulator, in which suppression of conductivity occurs as a result of interactions rather than a filled Bloch band[1]. Proximity to the Mott insulating phase in fermionic systems is the origin of many intriguing phenomena in condensed matter physics[2], most notably high-temperature superconductivity[3]. The Hubbard model[4], which encom-

the physics of strongly correlated systems. In an optical lattice three mutually perpendicular standing laser waves create a periodic potential for the atoms. The kinetics of the atoms is determined by their tunnelling rate between neighbouring lattice sites, and the interaction is due to interatomic collisions occurring when two atoms are on the same site. In a gas of fermions in different spin states this collisional interaction can be widely tuned through a Feshbach resonance with-

Observation of the phase transition

Typical experimental setup, as in our group



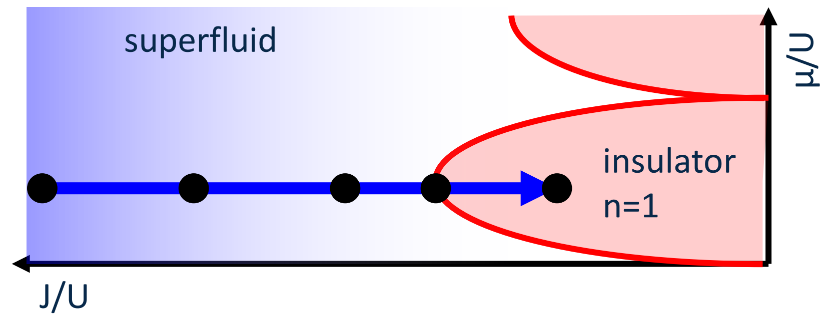
Lattice depth — Tunneling **J**

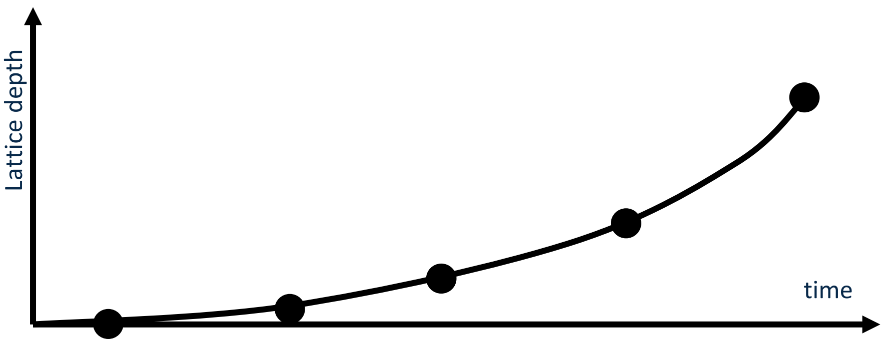
Scattering length — Interaction **U**

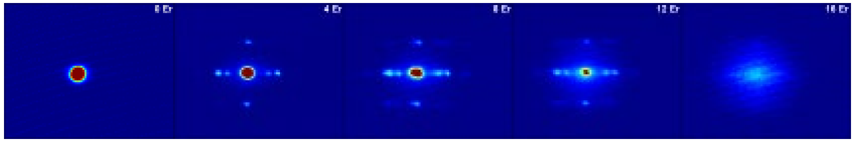
Dipole trap — External potential **ε**

Measurement method

Probe coherence by TOF measurements



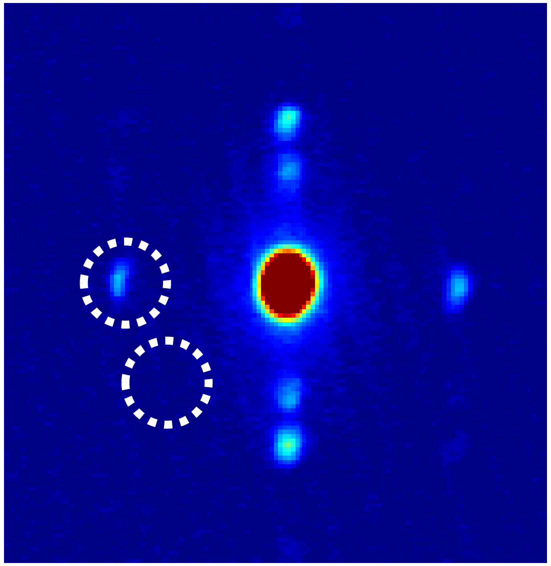


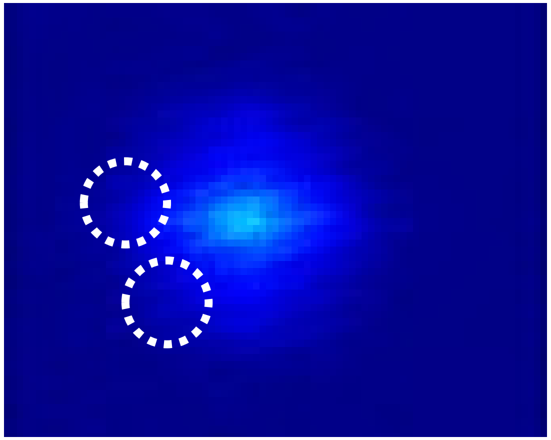


superfluid — Mott insulator

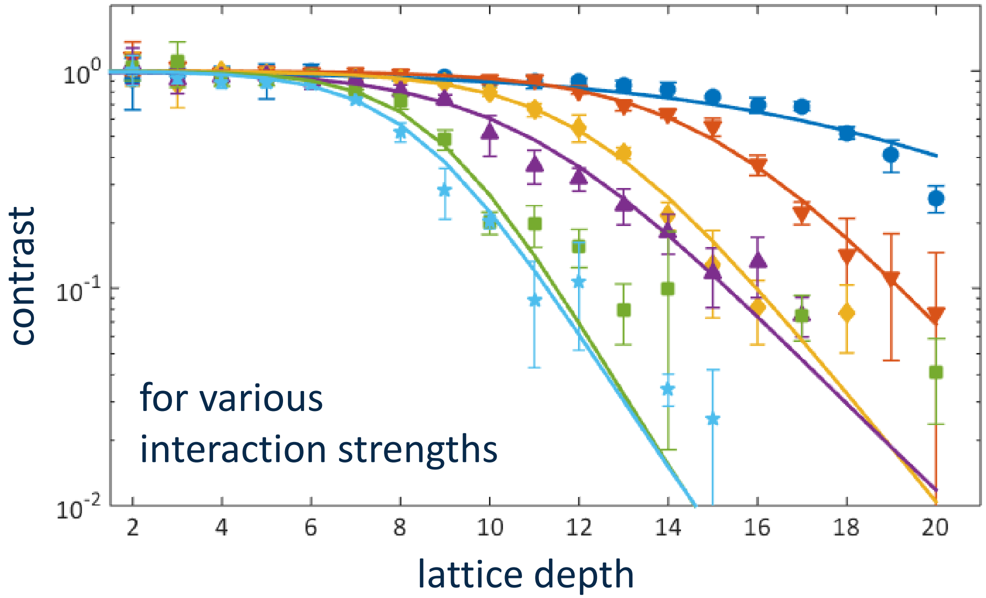
Observation of the phase transition

Observable: Contrast



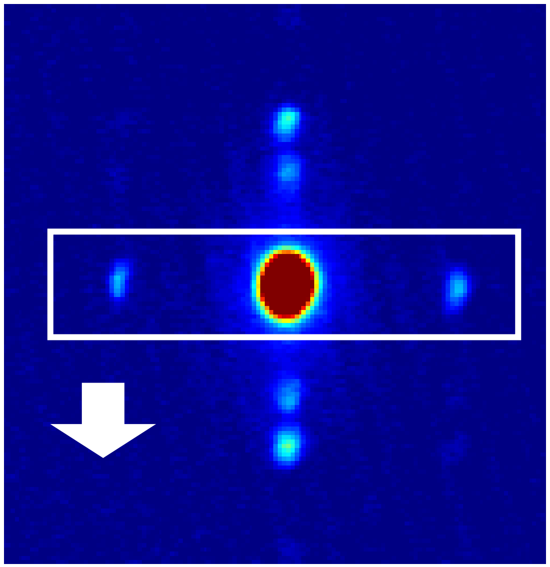


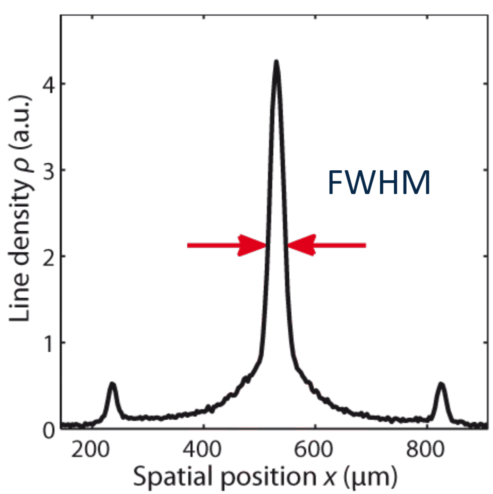
Results



Observation of the phase transition

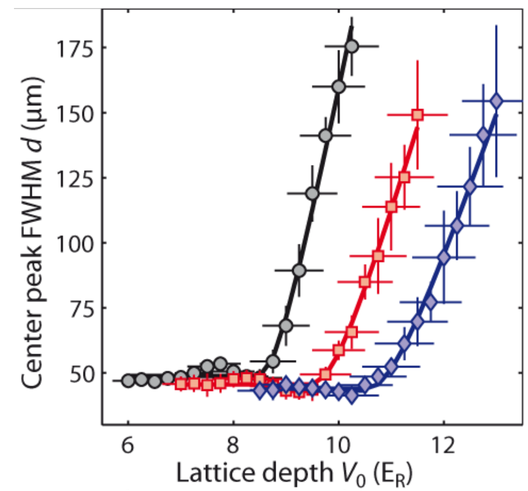
Observable: Width



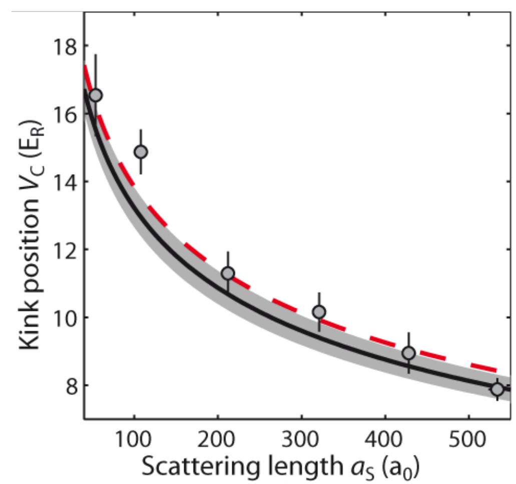


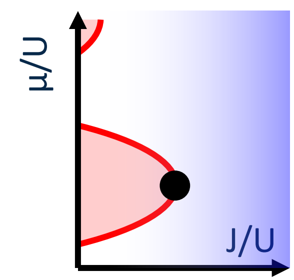
Results

'Kink' in FWHM



Phase transition point



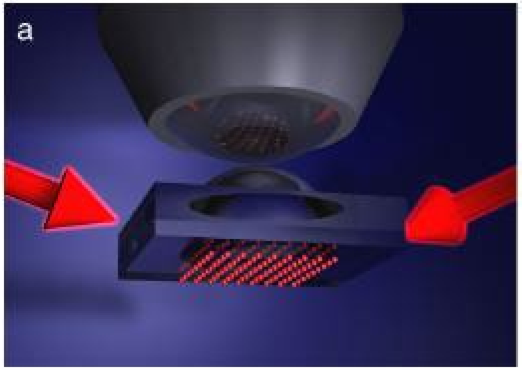


Mark *et al.* Phys. Rev. Lett. 107, 175301 (2011)

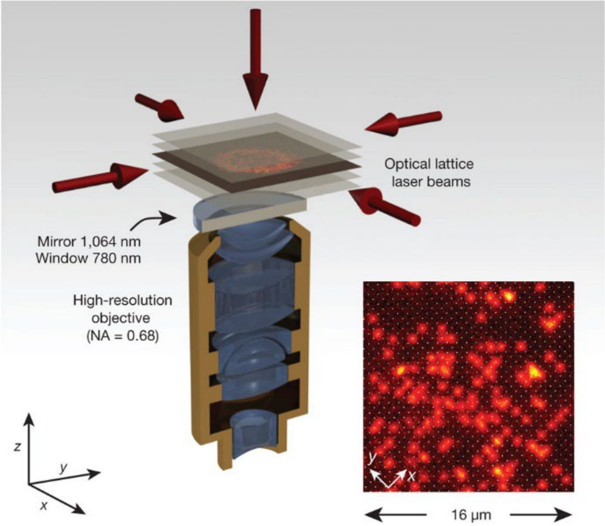
Observation of the phase transition

Quantum-gas microscope

System

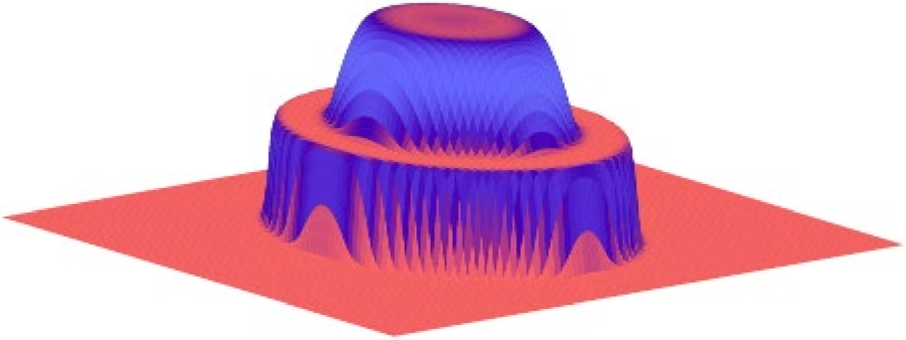


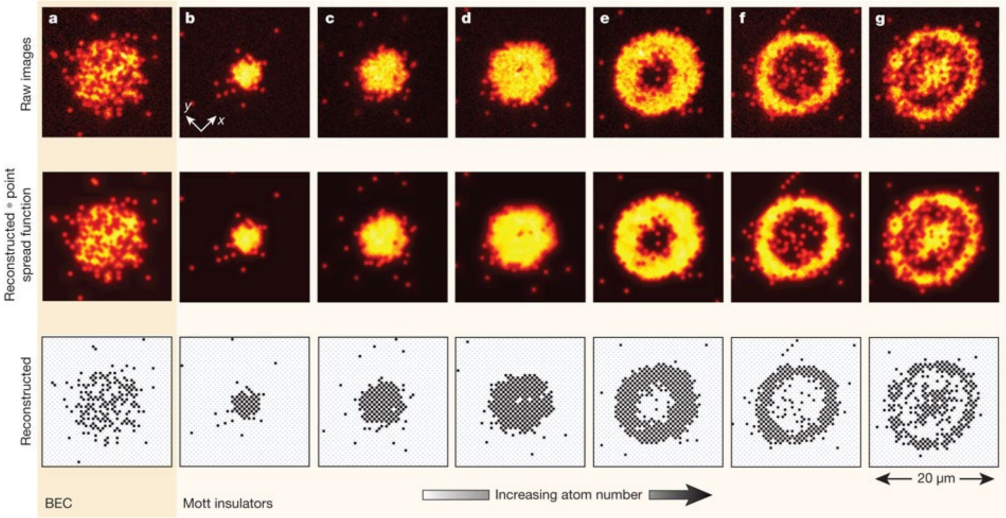
Greiner group @ Harvard



Bloch group @ Munich

'wedding cake structure'



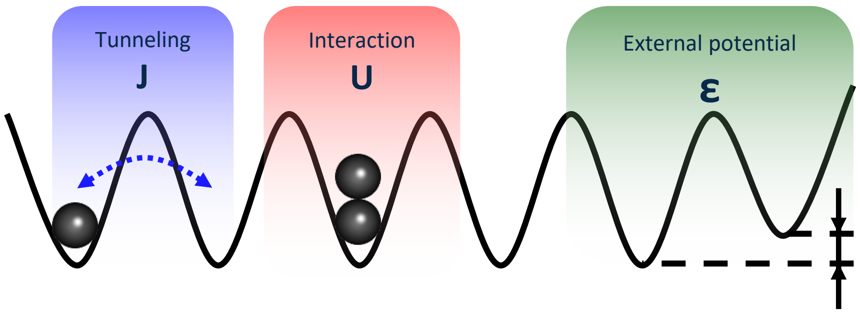


Sherson *et al.* Nature 467, 68-72 (2010)

Bose-Hubbard and beyond

The standard Bose-Hubbard model

$$\hat{H} = -J \sum_{<i,j>} \hat{a}_i^\dagger \hat{a}_j + \sum_i \frac{U}{2} \hat{n}_i (\hat{n}_i - 1) + \sum_i \epsilon_i \hat{n}_i$$



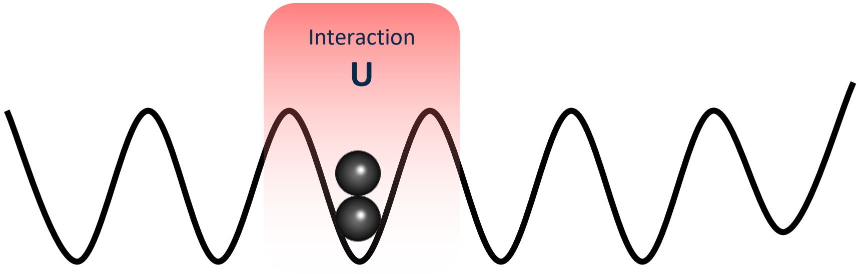
Is this enough to describe the systems accuratly?

Interaction term		Multibody interactions
Tunneling term	+	Density-induced tunneling
Potential shifts		Nearest-neighbor interactions

Investigating onsite interactions

The standard Bose-Hubbard model

$$\widehat{H} = -J \sum_{<i,j>} \widehat{a}_i^\dagger \widehat{a}_j + \sum_i \frac{U}{2} \widehat{n}_i \left(\widehat{n}_i - 1\right) + \sum_i \epsilon_i \widehat{n}_i$$

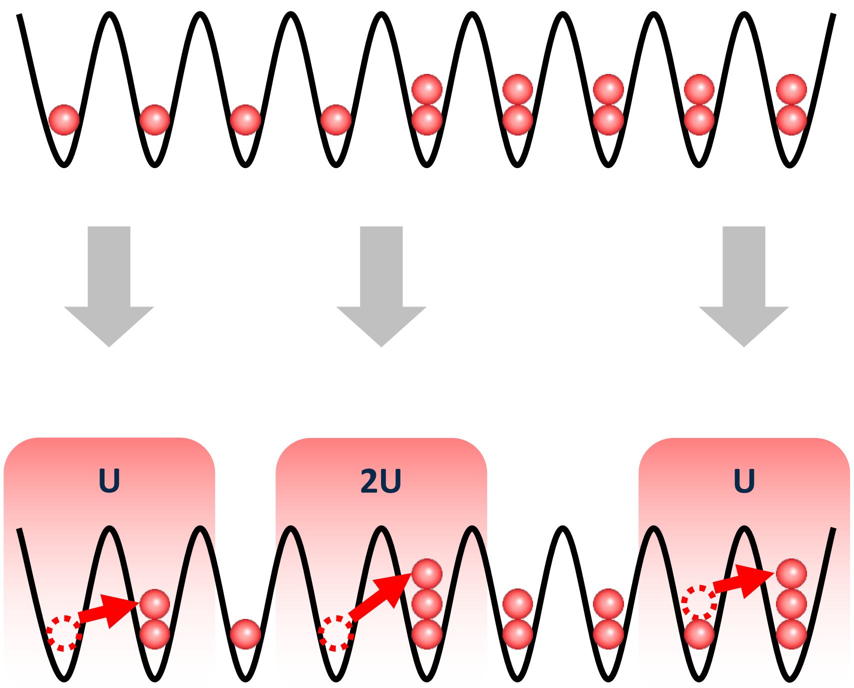


Let's have a closer look at on-site contact interactions!

Investigating onsite interactions

MI excitation spectrum

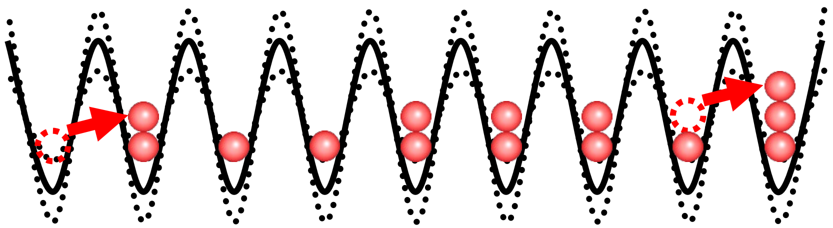
Elementary MI excitations



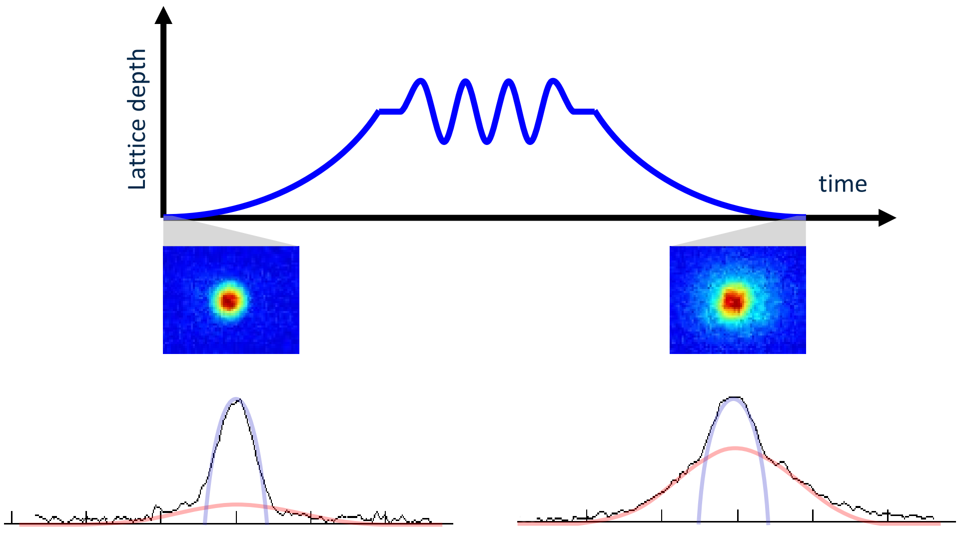
Measurement method

$\nu = U/h$

Amplitude modulation

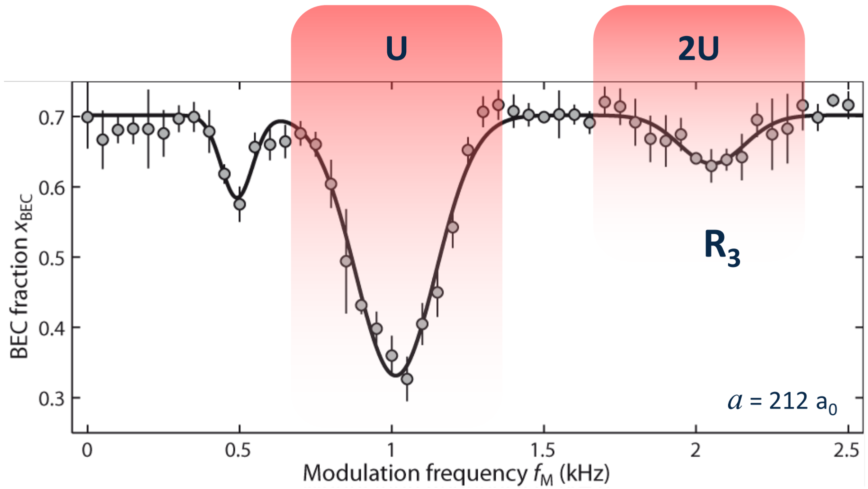


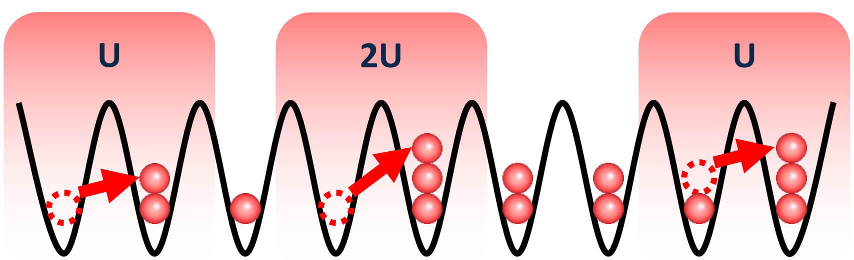
Experimental sequence



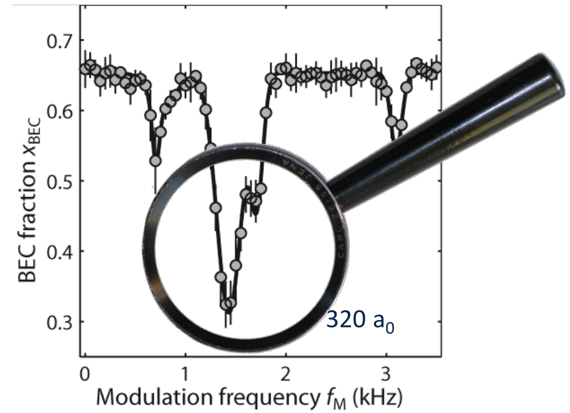
Investigating onsite interactions

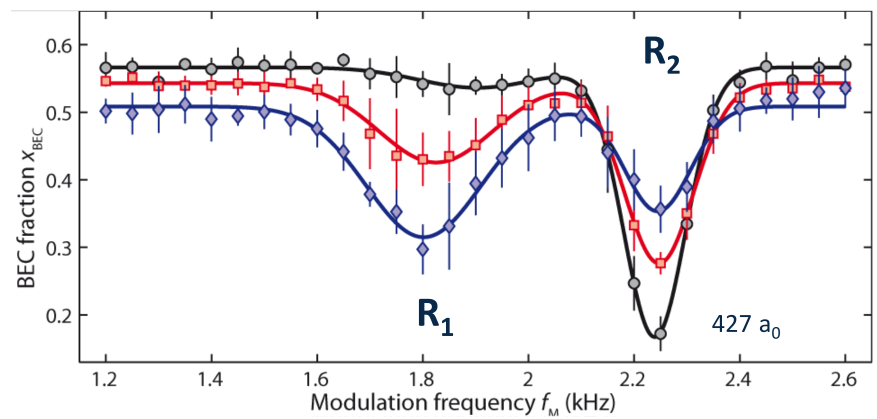
Results





Resonance splitting





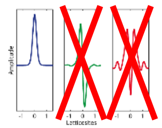
Density dependence

Mark *et al.,* Phys. Rev. Lett. 107, 175301 (2011)

Investigating onsite interactions

Approximations

Bloch bands



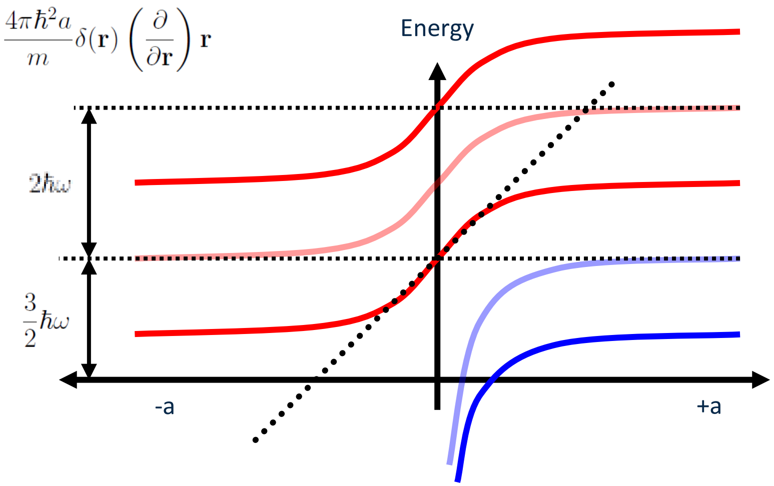
Interaction potential

$$g(\mathbf{x},\mathbf{x}') = \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{x}-\mathbf{x}')$$

Invalid for strong interactions

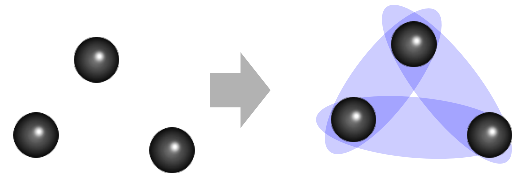
Two particles

$$g(\mathbf{r}) = \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r})\left(\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{r}$$



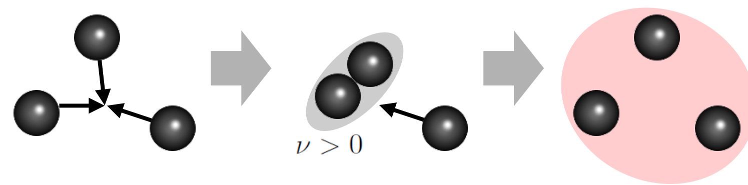
Busch *et al.*, Found. of Physics 28, 549 (1998)
Schneider *et al.*, Phys. Rev. A 80, 013404 (2009)
Büchler *et al.*, Phys. Rev. Lett. 104, 090402 (2010)

Three particles



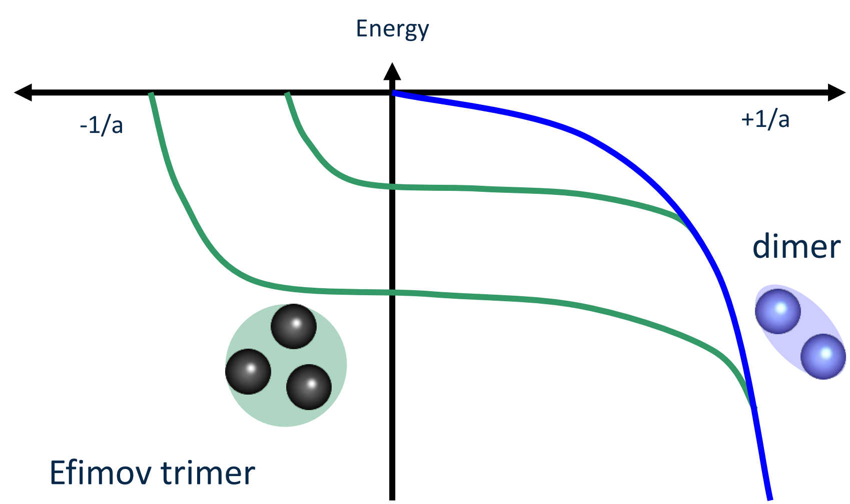
3x two-particle interactions

Effective three-body interactions



Johnson *et al.*, New J. Phys. 11, 093022 (2009)

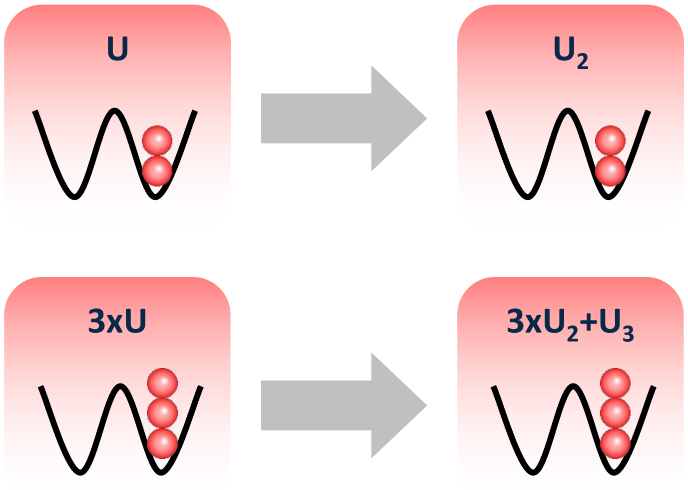
Efimov physics



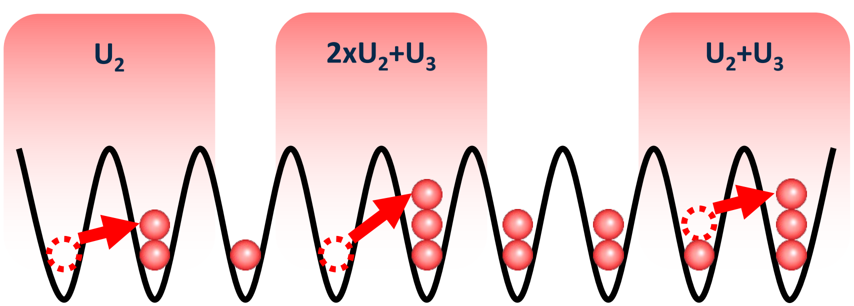
Kraemer *et al.*, Nature 440, 315 (2006)

Investigating onsite interactions

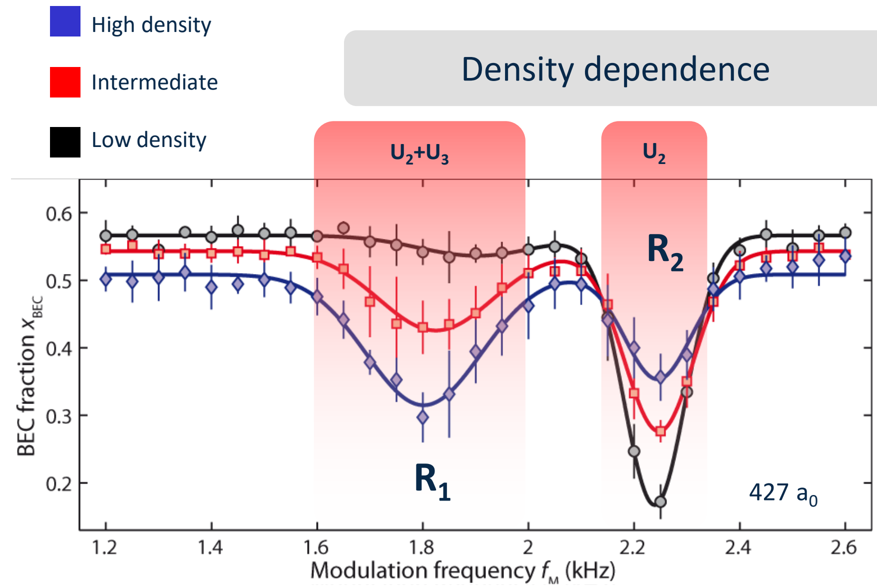
Expectation

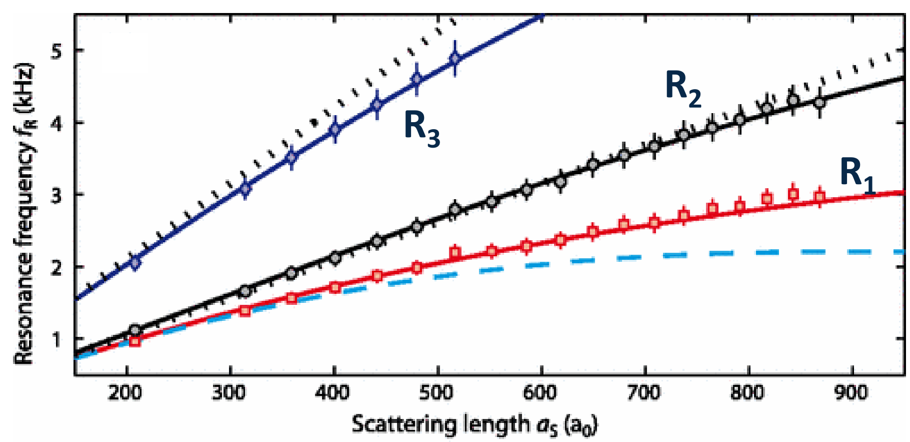


Thus:



Measurement



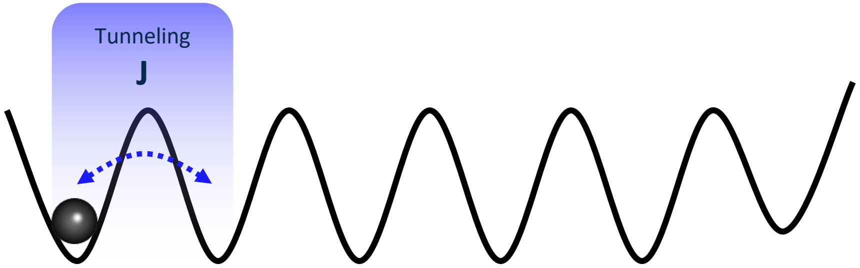


Mark *et al.*, Phys. Rev. Lett. 107, 175301 (2011)

Investigating tunneling dynamics

The standard Bose-Hubbard model

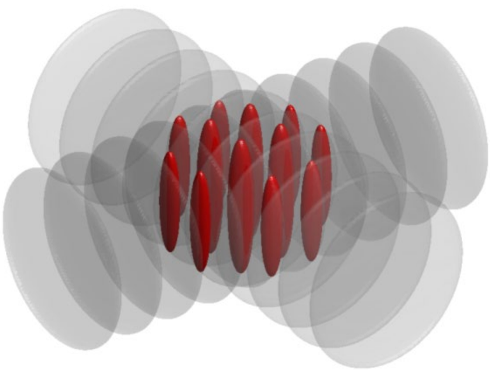
$$\widehat{H} = -J \sum_{<i,j>} \widehat{a}_i^\dagger \widehat{a}_j + \sum_i \frac{U}{2} \widehat{n}_i \left(\widehat{n}_i - 1 \right) + \sum_i \epsilon_i \widehat{n}_i$$



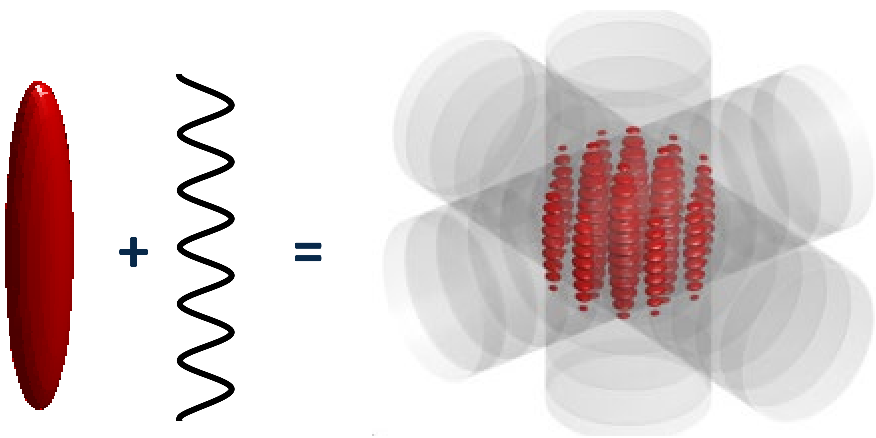
Let's have a closer look on tunneling dynamics!

Investigating tunneling dynamics

Going into 1D

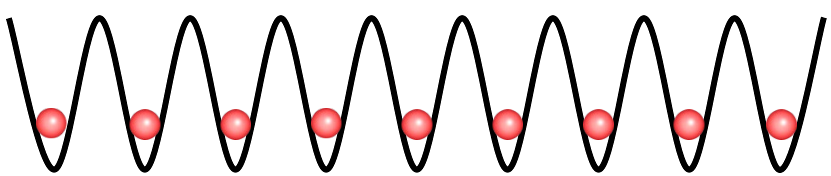


1D tubes + lattice



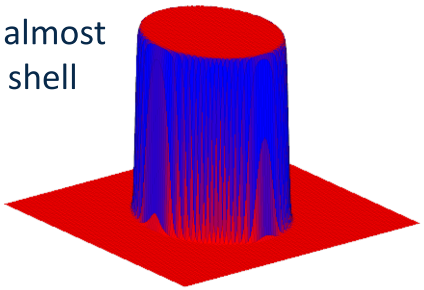
Starting point

Each tube is initially in a 1D Mott insulator configuration with one atom per lattice site



Loading distribution

3D Mott insulator with an almost pure singly occupied Mott shell

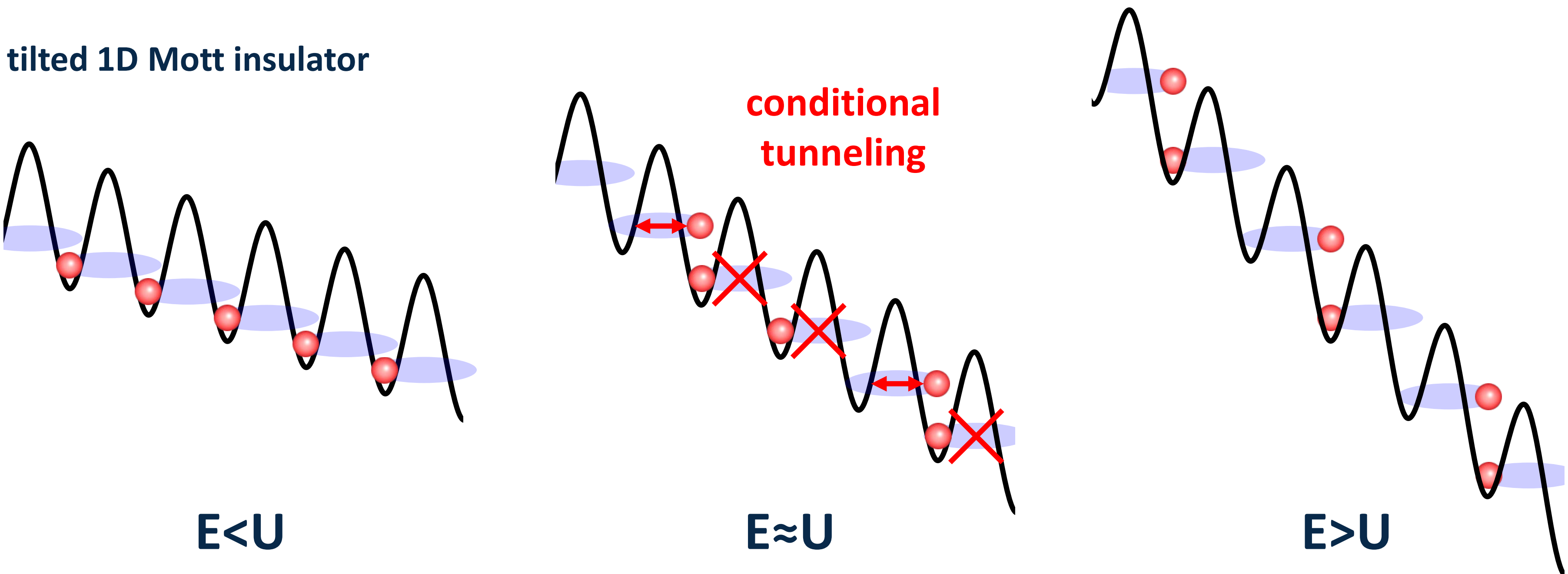


About 2000 tubes are populated with up to 60 atoms per tube

Investigating tunneling dynamics

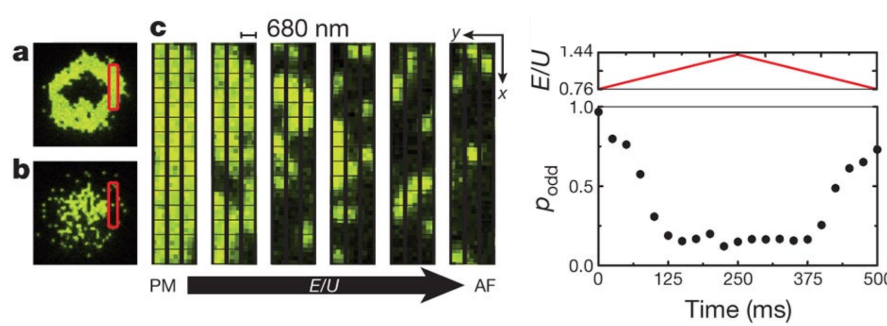
Maps to a spin model: S. Sachdev *et* al., Phys. Rev. B 66, 075128 (2002)

tilted 1D Mott insulator



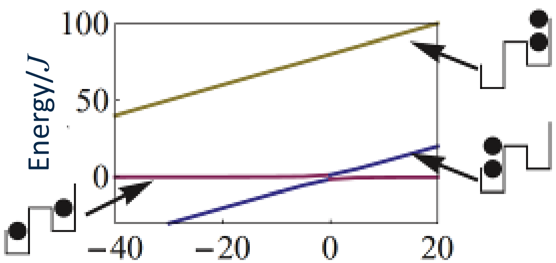
Experiment with a quantum-gas microscope: J. Simon *et* al., Nature 472, 307 (2011)

Ising-type phase transition: PM-AFM



Quench near the critical point

Double-well system



On resonance: $f_{osc} = 2\frac{\sqrt{2}J}{2\pi\hbar}$

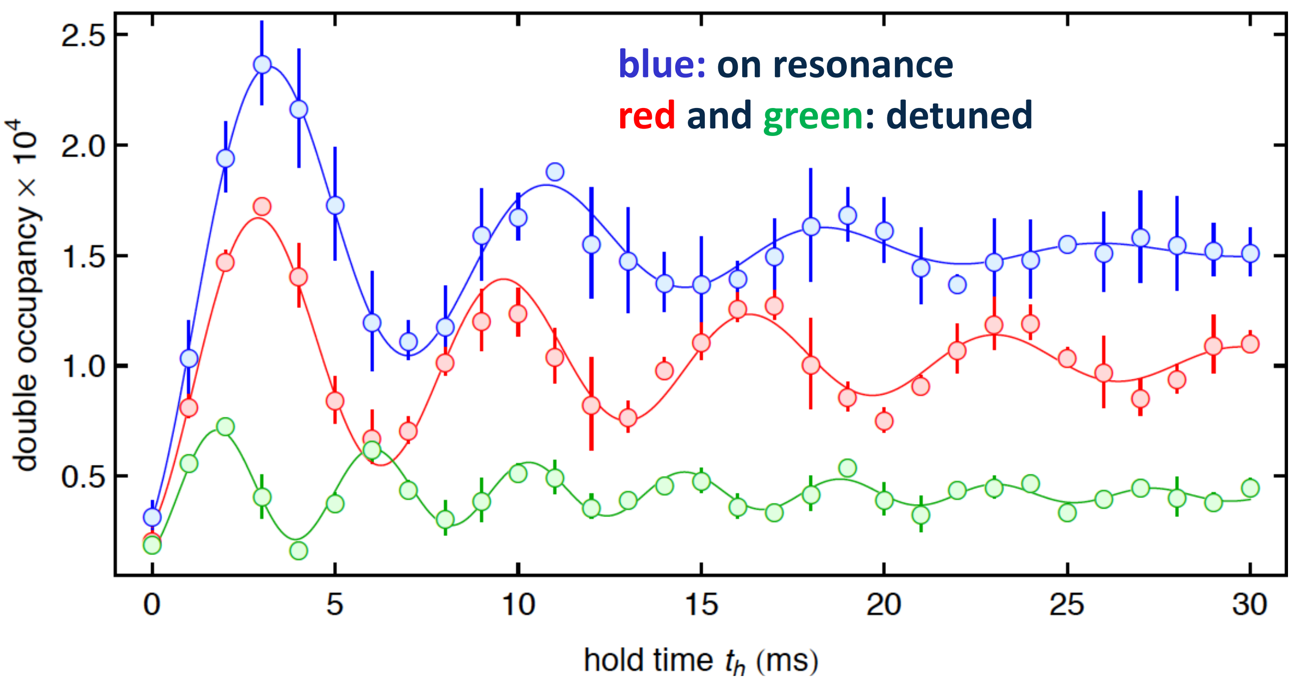
With detuning: $f_{osc} \sim (E-U)^2$

Theory: P. Rubbo *et al.*, Phys. Rev. A 84, 033638 (2011)

Measurement

- Change tilt within 1ms
- Wait hold time
- Change tilt back
- Freeze the system (lattice depth)
- Measure double occupancy

Experiment: F. Meinert *et al.*, Phys. Rev. Lett. 111, 053003 (2013)



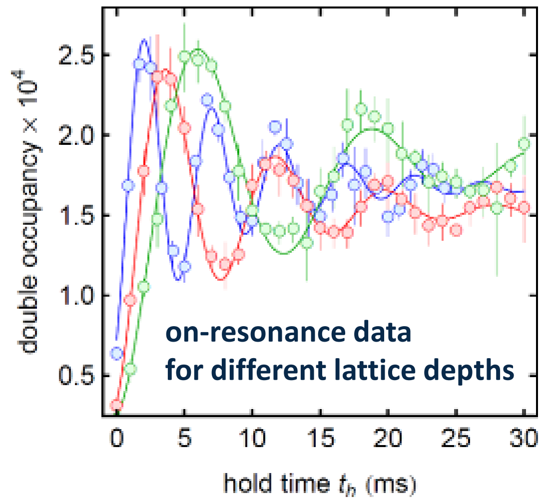
Investigating tunneling dynamics

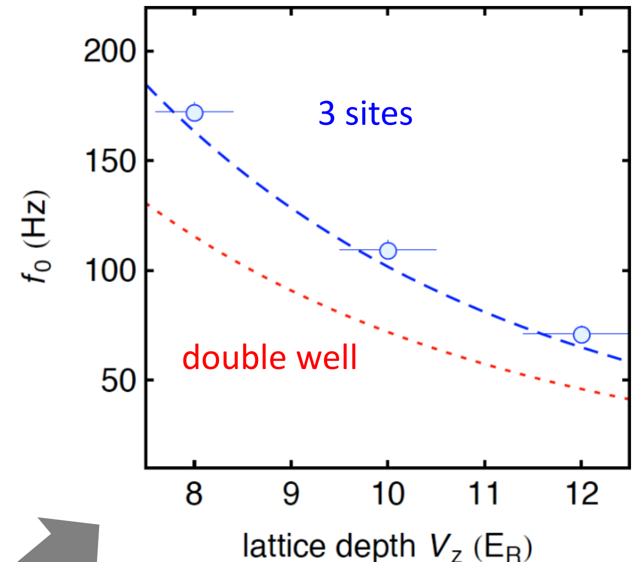
Multibody effects

Oscillation frequency

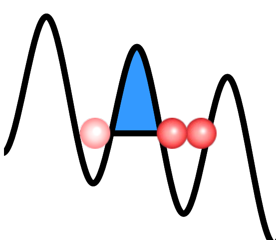
Double well on resonance:

$$f_{osc} = 2\frac{\sqrt{2}J}{2\pi\hbar}$$

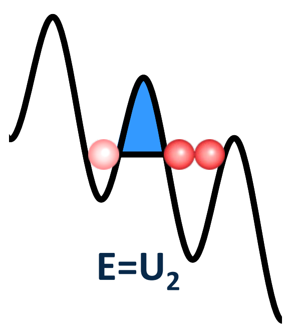




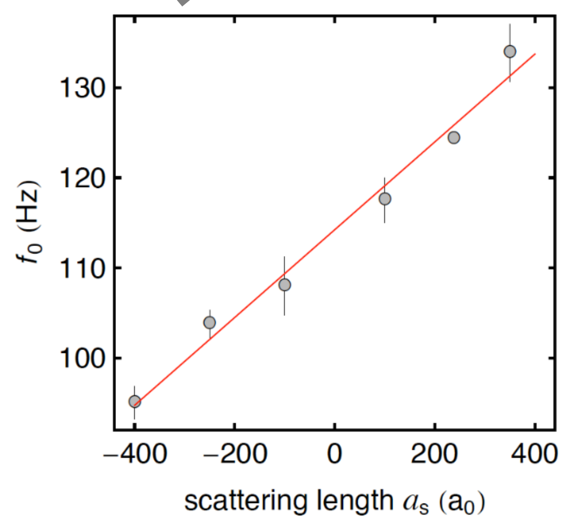
Effect of interactions



E=U_1

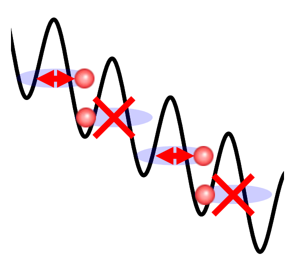


E=U_2



Effect of constraint

$$f_{osc} = \sqrt{2}\cdot 2\frac{\sqrt{2}J}{2\pi\hbar} \quad \text{(N=4)}$$



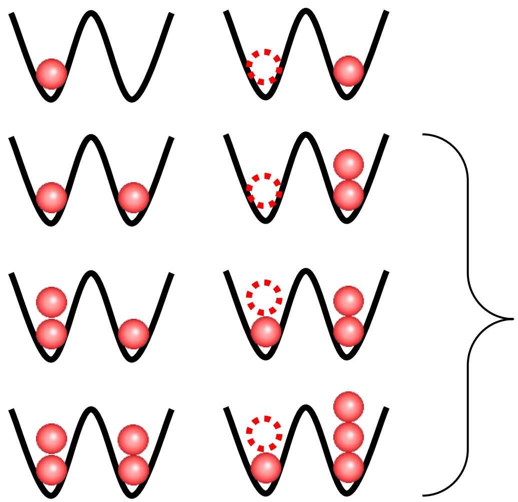
D.-S. Lühmann *et* al., New J. Phys. 14 033021 (2012)

U. Bissbort *et* al., Phys. Rev. A 86, 023617 (2012)

Investigating tunneling dynamics

Tunneling modified by interactions

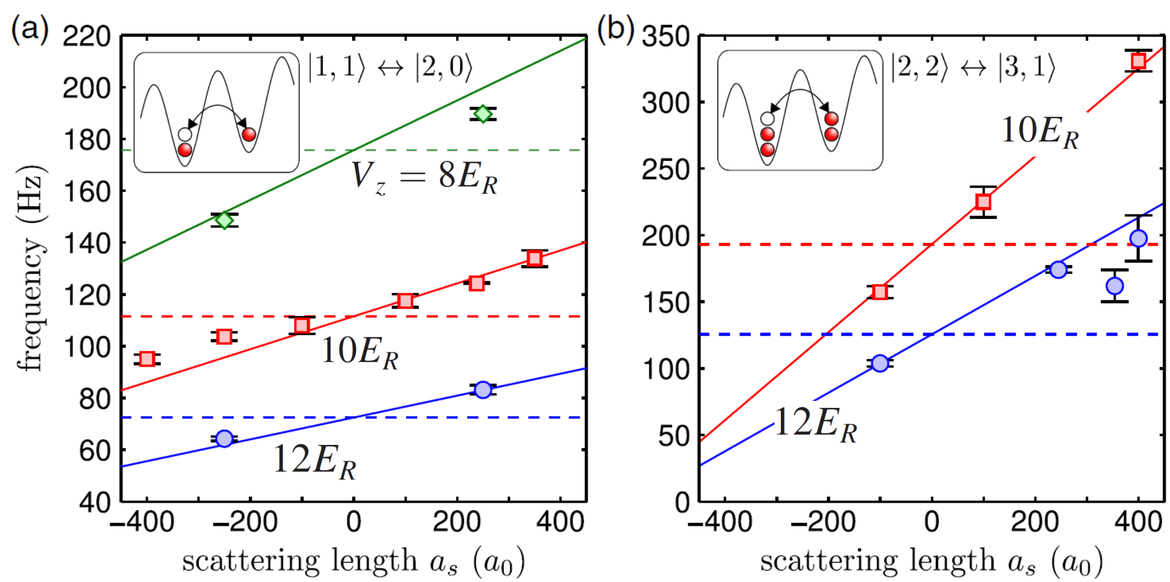
density-induced tunneling



$$J = -\int dx w_0(x) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_0 \sin^2(kx)\right) w_0(x-d)$$

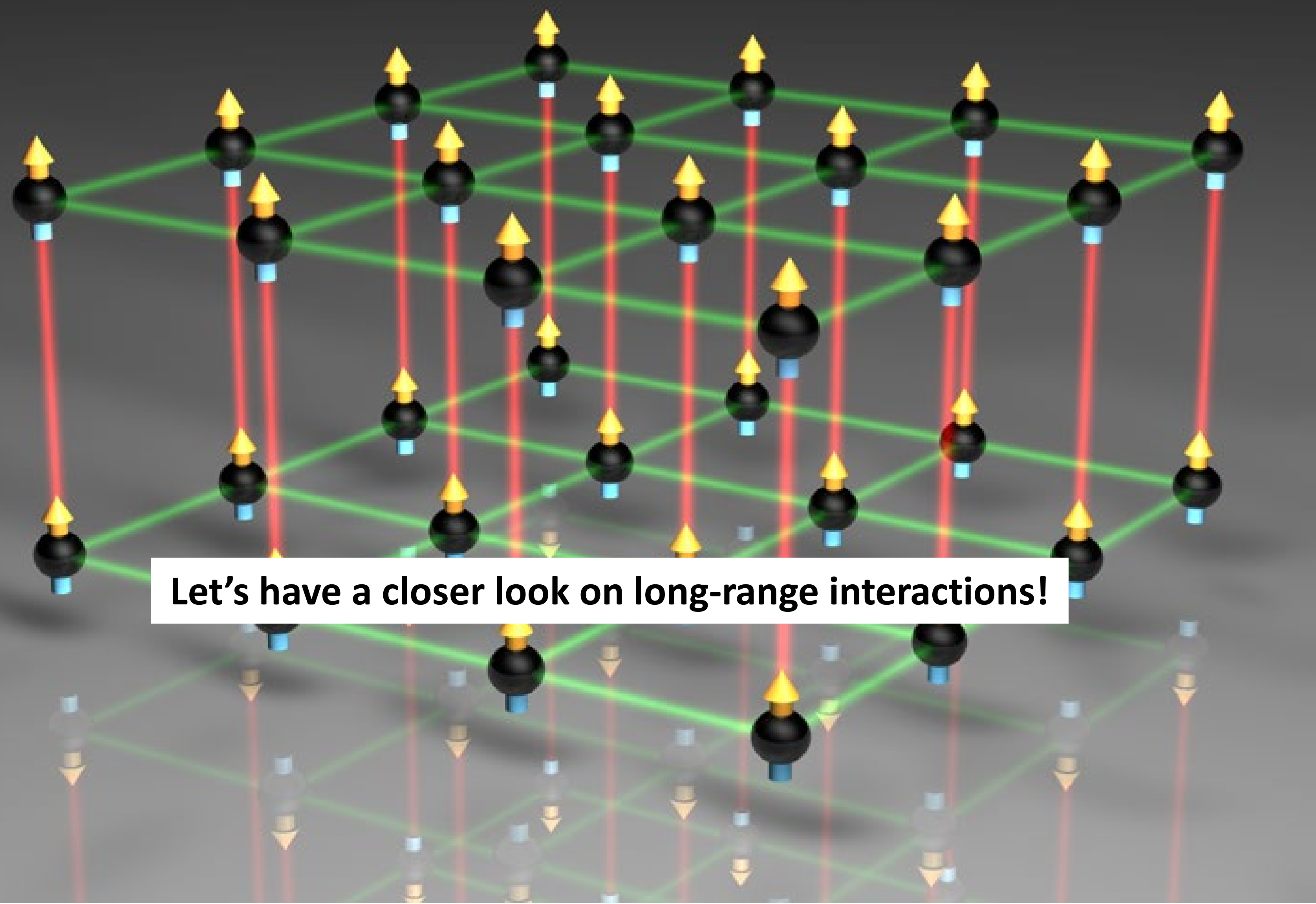
$$\Delta J = -\frac{4\pi\hbar^2}{m}\int d^3 r w^*(\mathbf{r}-\mathbf{d}) w^*(\mathbf{r}) w^2(\mathbf{r})$$

$$J_{\mathrm{tot}} = J + (n_i + n_j - 1) a_s \Delta J$$



Experiment: Jürgensen *et al.*. Phys. Rev. Lett. 113, 193003 (2014)

Long-range interactions extensions



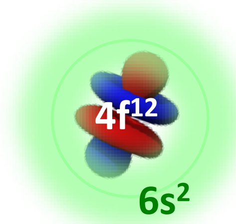
Let's have a closer look on long-range interactions!

Erbium properties

*Lanthanide series	lanthanum 57 La 138.91	cerium 58 Ce 140.12	praseodymium 59 Pr 140.91	neodymium 60 Nd 144.24	promethium 61 Pm [145]	samarium 62 Sm 150.36	europium 63 Eu 151.96	gadolinium 64 Gd 157.25	terbium 65 Tb 158.93	dysprosium 66 Dy 162.50	holmium 67 Ho 164.93	erbium 68 Er 167.26	thulium 69 Tm 168.93	ytterbium 70 Yb 173.04
**Actinide series	actinium 89 Ac [227]	thorium 90 Th 232.04	protactinium 91 Pa 231.04	uranium 92 U 238.03	neptunium 93 Np [237]	plutonium 94 Pu [244]	americium 95 Am [243]	curium 96 Cm [247]	berkelium 97 Bk [247]	californium 98 Cf [251]	einsteinium 99 Es [252]	100 Fm [257]	mendelevium 101 Md [258]	nobelium 102 No [259]

Bose-condensed   Laser-cooled

Submerged-shell structure



- Large electronic orbital angular momentum (L=5)

  Anisotropic van der Waals interaction

- Large mass and large magnetic moment of 7 μ_B

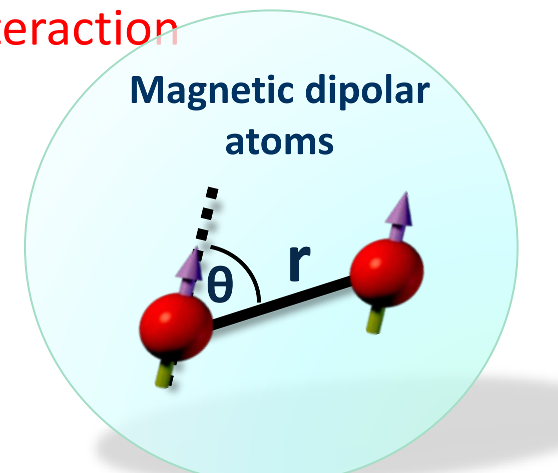
  Dipole-dipole interaction comparable to s-wave interaction

DIPOLE-DIPOLE INTERACTION (DDI)

$$U_{\mathrm{dd}} = \frac{C_{\mathrm{dd}}}{4\pi} \frac{1 - 3\cos^2\theta}{r^3}$$

anisotropic

long range



$$C_{dd} = \mu_0 \mu^2$$

Review articles:

T. Lahaye et al., Rep. Prog. Phys. 72, 126401 (2009).

M.A. Baranov, Phys. Rep. 464, 71 (2008).

The extended Bose-Hubard model

The dipolar interaction

$$\hat{H}_{\mathrm{dd}} = \frac{1}{2}\int \mathbf{d}^3 r_1 \mathbf{d}^3 r_2 \hat{\psi}^\dagger(\mathbf{r}_1)\hat{\psi}^\dagger(\mathbf{r}_2) U_{\mathrm{dd}}(\mathbf{r}_1 - \mathbf{r}_2)\hat{\psi}(\mathbf{r}_1)\hat{\psi}(\mathbf{r}_2)$$

$$U_{\mathrm{dd}} = \frac{C_{\mathrm{dd}}}{4\pi}\frac{1 - 3\cos^2\theta}{r^3}$$

anisotropic

long range

Expansion in Wannier basis

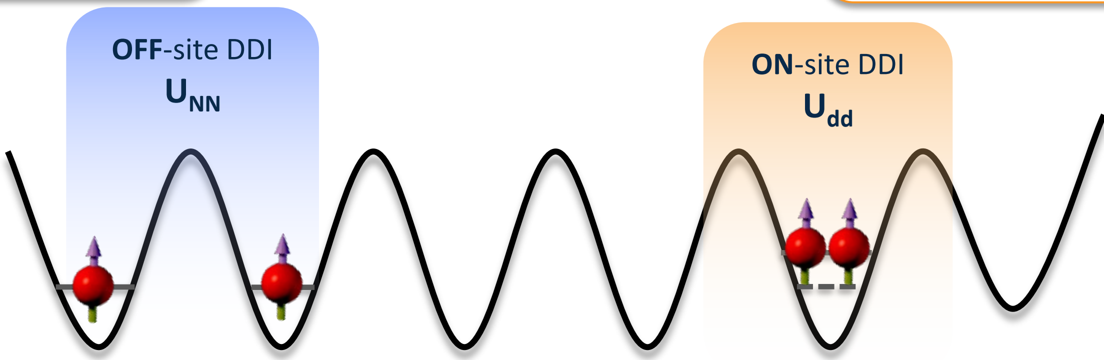
$$\hat{H}_{\mathrm{dd}} = \sum_{i,j,k,l} \frac{V_{ijkl}}{2} \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l$$

OFF-site dipole-dipole interaction (NNI)

$$\hat{H}_{\mathrm{dd}}^{\mathrm{NN}} = \sum_{i \neq j} \frac{V_{ij}}{2} \hat{n}_i \hat{n}_j$$

ON-site dipole-dipole interaction

$$\hat{H}_{\mathrm{dd}}^{\mathrm{on-site}} = \sum_{i} \frac{U_{\mathrm{dd}}}{2} \hat{n}_i (\hat{n}_i - 1)$$

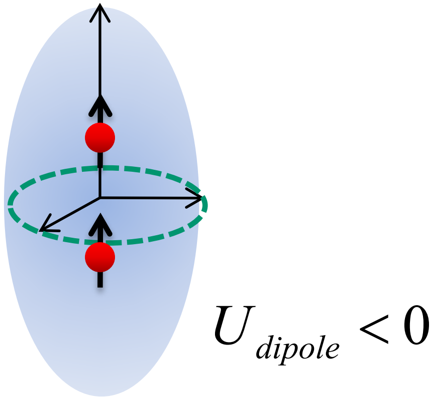


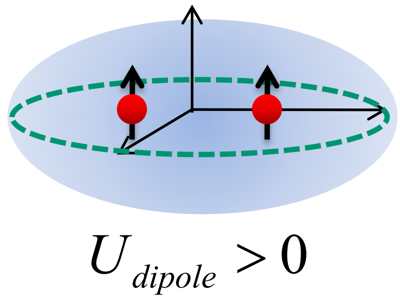
Reviews:

O. Dutta *et al.*, *Reports on Progress in Physics* 78, 066001 (2015)

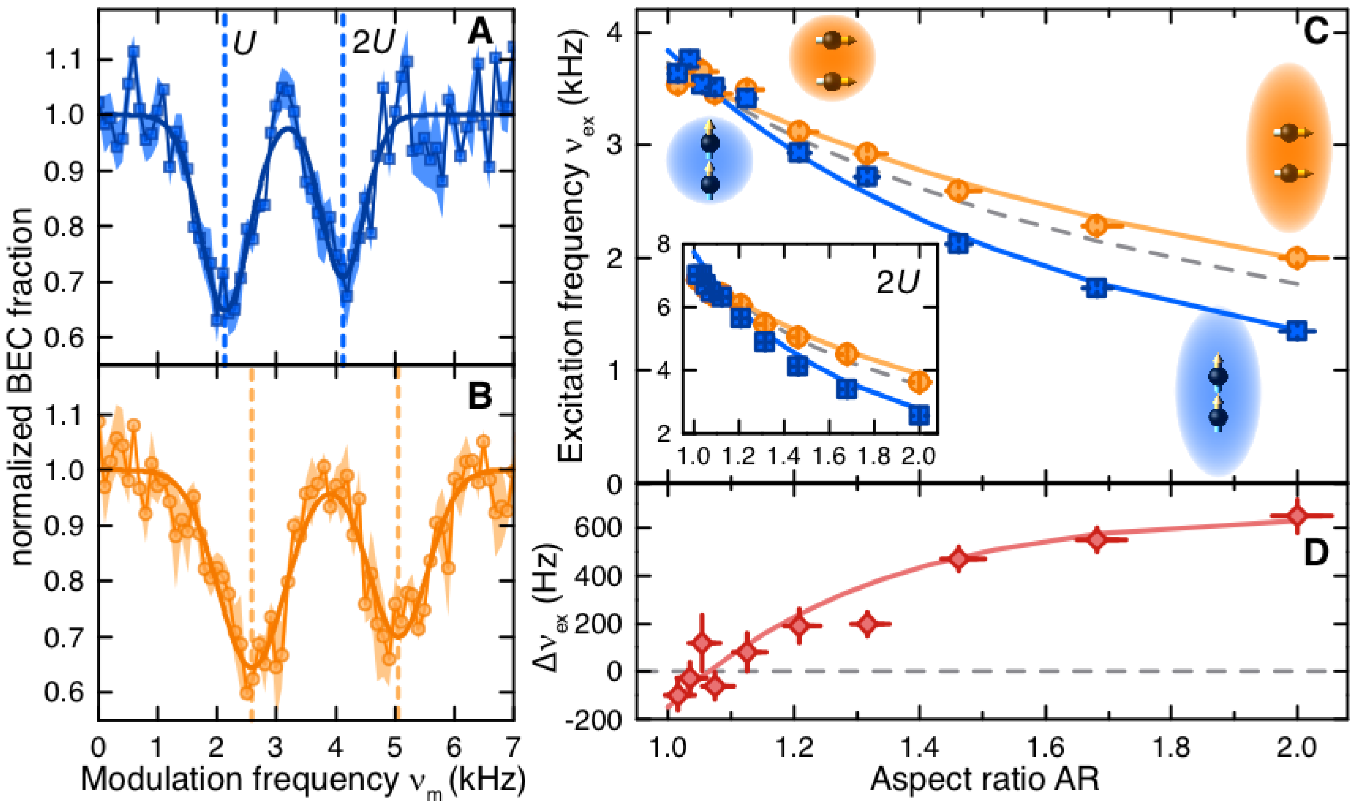
C. Trefzger et al., *Journal of Physics B: Atomic, Molecular and Optical Physics* 44, 193001 (2011)

Anisotropic on-site interactions



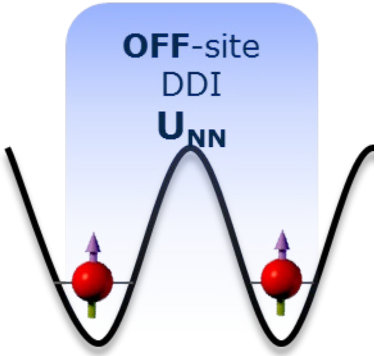


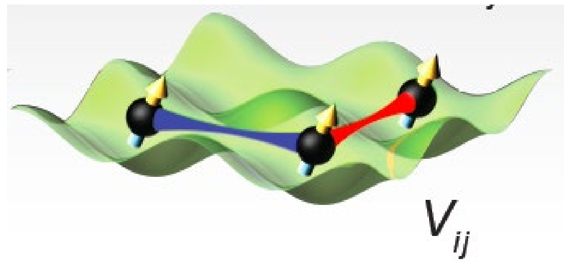
on-site interaction can be shaped by changing the lattice confinement



Experiment: S. Baier, et al. Science 352, 201 (2016)

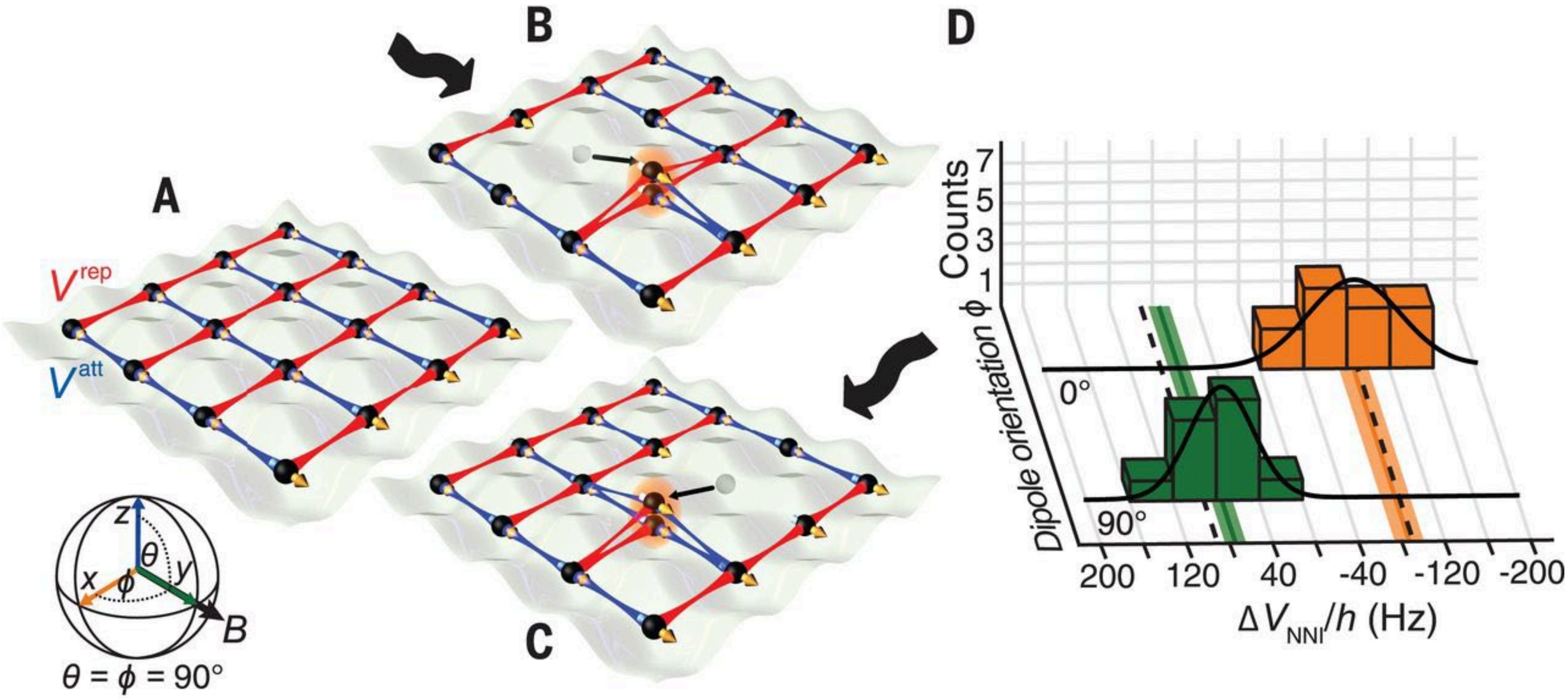
Nearest-neighbour interactions





BUT

V_{NNI} 30 Hz vs U_{ONSITE} 3 kHz



$$\Delta = \Delta E^{(x)} - \Delta E^{(y)} = \Delta U_{rep} - \Delta U_{att}$$

Experiment: S. Baier, et al. Science 352, 201 (2016)

Structure of the lecture

Basic introduction to Hubbard models

Interactions and tunneling in the BHM

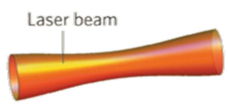
Basic introduction into 1D systems

Strongly interacting Bosons in 1D systems

One-dimensional systems

"Classical" approximation	"intermediate" (confinement dominated)	"quasi" one-dimensional	"purely" one-dimensional
Problem depends on the motion in only one direction.	Motion hindered by discrete energy levels	Transversal motion completely "frozen out"	

("*ignore*" other directions)

e.g. 

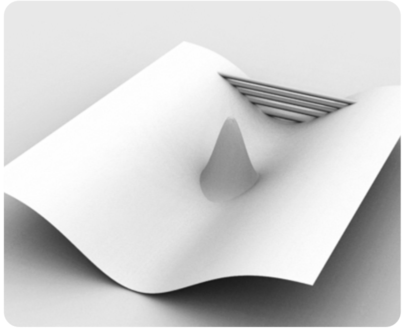
elongated traps

fixed Ansatz for radial part of wave function

$$\psi(\mathbf{r}) = \psi_r(r,\sigma)\psi_z(z)$$

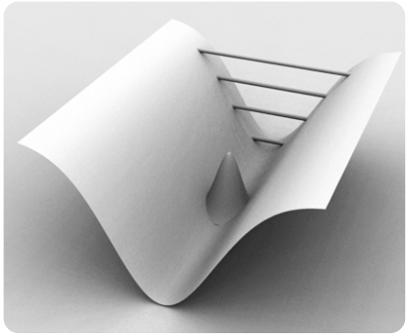
$k_B T \sim E_{gap}$

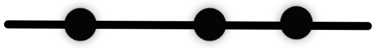
$\mu \sim E_{gap}$



$k_B T << E_{gap}$

$\mu << E_{gap}$





- mathematical concept
- one spatial coordinate
- Hamilton operator

(+contact interaction)

$$H = \frac{p_z^2}{2m} + g\,\delta(z)\,\frac{\partial}{\partial z}$$

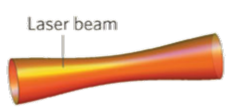
One-dimensional systems

"Classical" approximation	"intermediate" (confinement dominated)	"quasi" one-dimensional	"purely" one-dimensional

"Classical" approximation

Problem depends on the motion in only one direction.

(*"ignore" other directions)*

e.g. 

elongated traps

fixed Ansatz for radial part of wave function

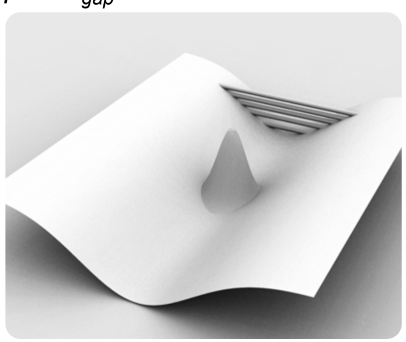
$$\psi(\mathbf{r}) = \psi_r(r,\sigma)\psi_z(z)$$

"intermediate" (confinement dominated)

Motion hindered by discrete energy levels

$k_B T \sim E_{gap}$

$\mu \sim E_{gap}$

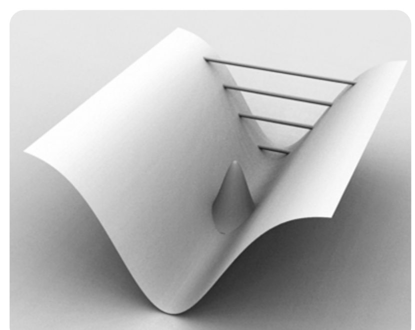


"quasi" one-dimensional

Transversal motion completely "frozen out"

$k_B T << E_{gap}$

$\mu << E_{gap}$



"purely" one-dimensional

- mathematical concept
- one spatial coordinate
- Hamilton operator

(+contact interaction)

$$H = \frac{p_z^2}{2m} + g\,\delta(z)\,\frac{\partial}{\partial z}$$

Interactions in one-dimensional systems

gamma parameter

$$\gamma = \frac{\text{interaction energy}}{\text{kinetic energy}} = \frac{m g_{1D}}{\hbar^2 n}$$

g_{1D} – coupling constant in 1D

n – 1D density in a uniform system

General properties

- weakly interacting system, $\gamma<1$
- strongly interacting system, $\gamma>1$
- counter-intuitive: more interacting for smaller density *n*

alpha parameter

Interaction parameter for system with harmonic confinement

$$\alpha = \frac{m g_{1D}\, a_{||}}{\hbar^2}$$

$a_{||}$ – harmonic oscillator length

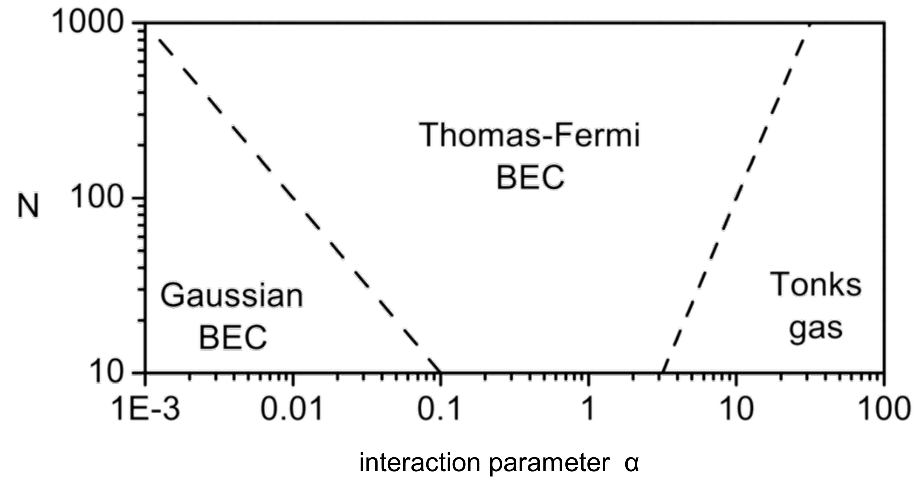
replace distance between atoms (1/n) by harmonic oscillator length

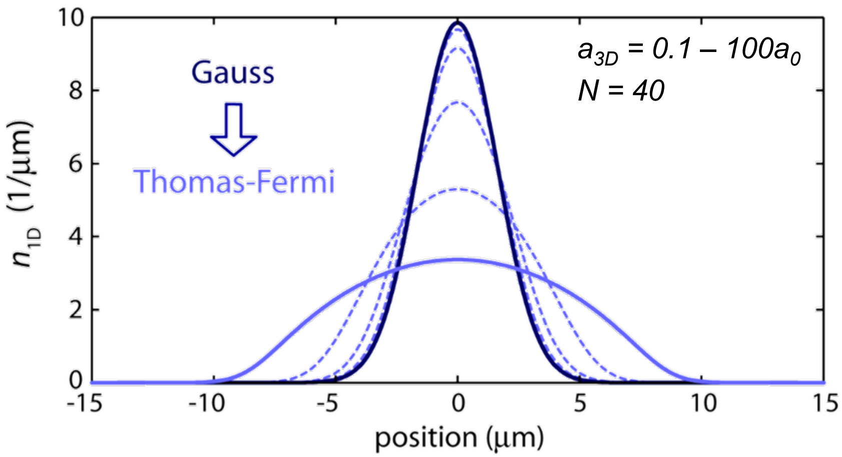
$$a_{||} = \sqrt{\frac{\hbar}{m\omega_{||}}}$$

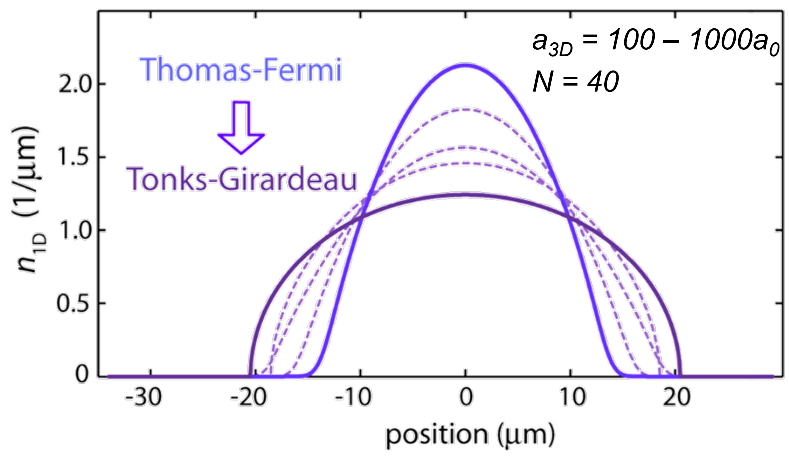
Phases in one-dimensional systems

Phase diagram T=0

D.S. Petrov et al.,
J. Phys. IV France 1 (2007)



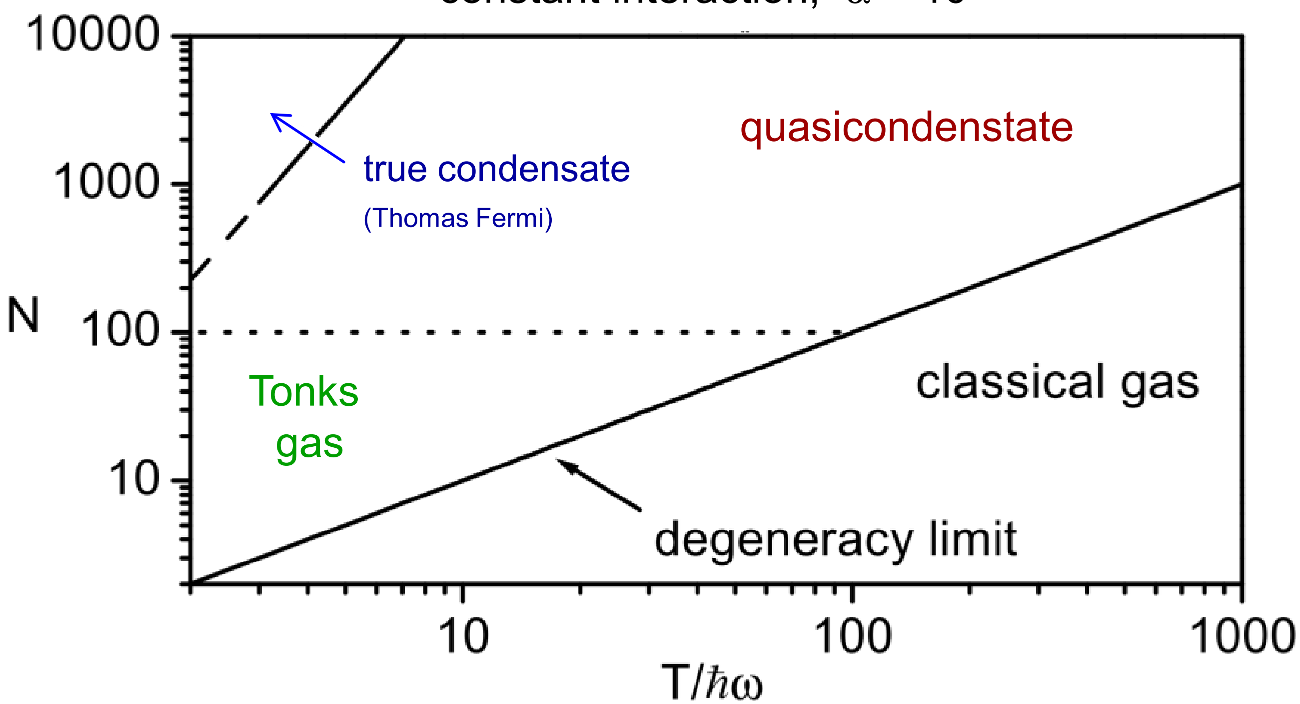




Phases in one-dimensional systems

Phase diagram at T≠0

constant interaction, $\alpha = 10$



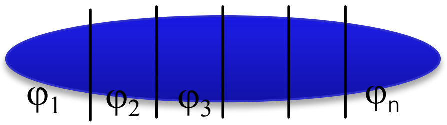
D.S. Petrov et al.,
J. Phys. IV France **1** (2007)

quasicondensate

fluctuating phase,

normal density distribution

$$\psi = \sqrt{n_0} e^{i\varphi(z,t)}$$



Lieb - Liniger model

Model: E. Lieb and W. Liniger,
Phys. Rev. **130**, 1605 (1963)

- bosons in uniform 1D system
- repulsive contact potential
- TG gas is exactly solvable

Hamilton operator:

$$H = -\sum_i \frac{\partial^2}{\partial x_i^2} + c\,\gamma \sum_{<i,j>} \delta(x_i - x_j)$$

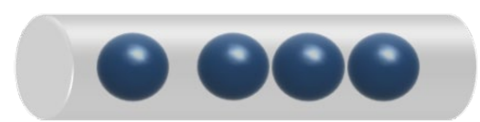
kinetic energy — interaction energy

c - constant
g - interaction strength

$$\gamma = \frac{m\, g_{1D}}{\hbar^2 n}$$

Ideal gas $\gamma = 0$
(non-interacting bosons)

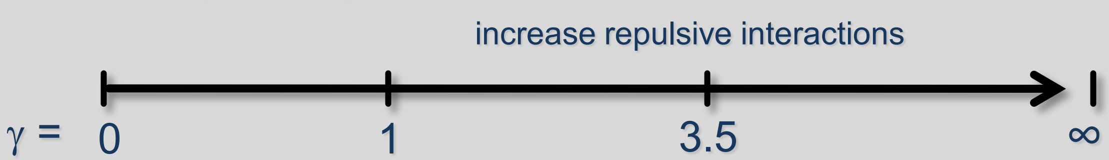
Tonks-Girardeau gas (TG)
(non-interacting fermions)
(hard spheres)
$\gamma \to \infty$



g - parameter

Interaction regimes of 1D quantum gases

bosons (Lieb-Liniger)



ideal gas

Tonks-Girardeau gas

One-dimensional systems

3D → **1D**

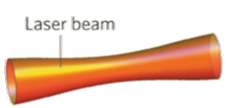
“Classical” approximation	“intermediate” (confinement dominated)	“quasi” one-dimensional	“purely” one-dimensional

“Classical” approximation

Problem depends on the motion in only one direction.

(“*ignore” other directions)*

e.g.



elongated traps

fixed Ansatz for radial part of wave function

$$\psi(\mathbf{r}) = \psi_r(r,\sigma)\,\psi_z(z)$$

“intermediate” (confinement dominated)

Motion hindered by discrete energy levels

$k_B T \sim E_{gap}$

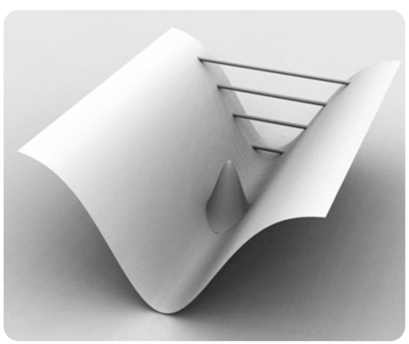
$\mu \sim E_{gap}$

“quasi” one-dimensional

Transversal motion completely “frozen out”

$k_B T \ll E_{gap}$

$\mu \ll E_{gap}$



“purely” one-dimensional

- mathematical concept
- one spatial coordinate
- Hamilton operator

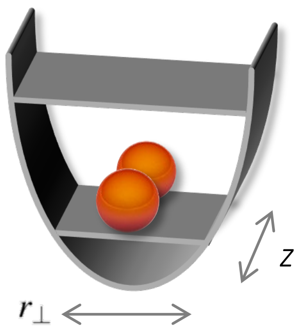
(+contact interaction)

$$H = \frac{p_z^2}{2m} + g\,\delta(z)\,\frac{\partial}{\partial z}$$

Scattering in quasi one-dimensional systems

M. Olshanii, PRL **81**, 938 (1998).

T. Bergeman *et al.*, PRL **91**,163201 (2003).



Hamiltonian: $H = H_{\mathrm{CM}} + H_{\mathrm{rel}}$

relative coordinates

$$H_{\mathrm{rel}} = \frac{p_z^2}{2\mu} + g_{3D}\,\delta(z, r_\perp)\frac{\partial}{\partial \mathbf{r}}\mathbf{r} + H_\perp(p_\perp, r_\perp)$$

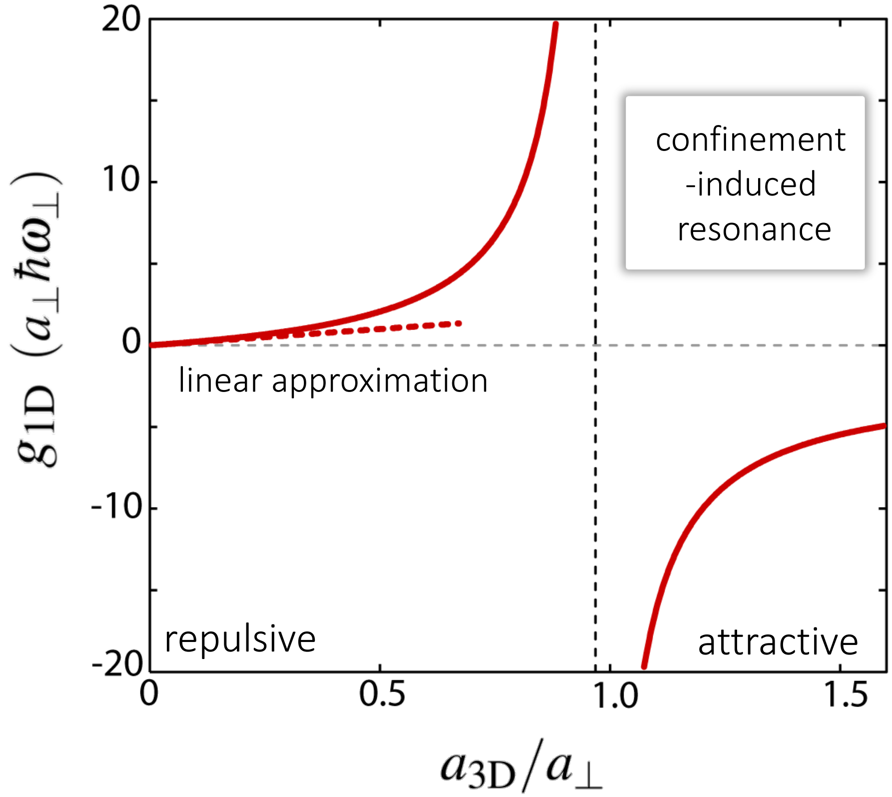
longitudinal — transversal

$$H_{1D} = \frac{p_z^2}{2\mu} + g_{1D}(a_{3D}, \omega_\perp)\,\delta(z)$$

1D coupling constant

$$g_{1D} = 2\hbar\omega_\perp a_{3D}\left(1 - C\frac{a_{3D}}{a_\perp}\right)^{-1}$$

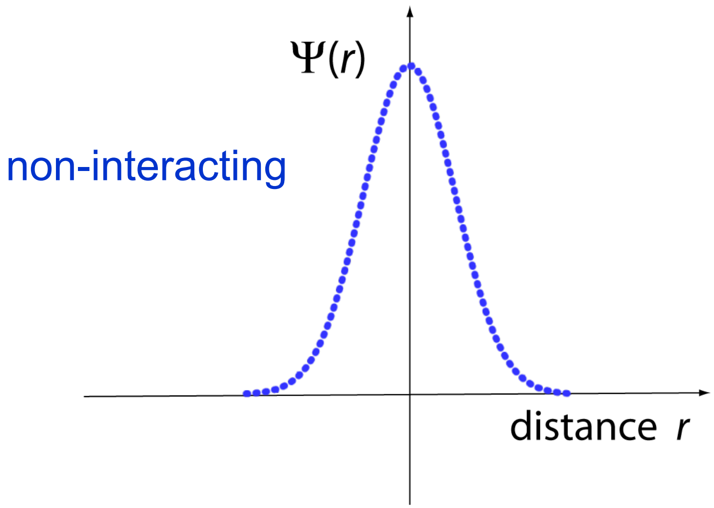
constant close to 1

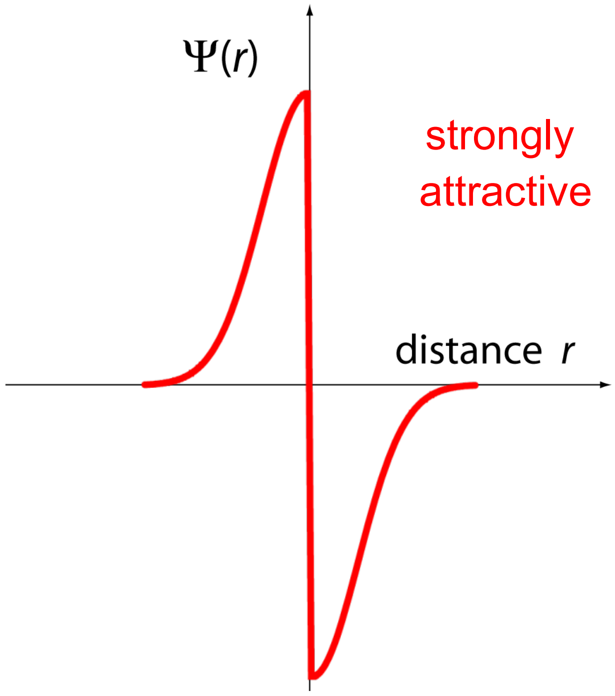


Bose-Fermi mapping

Identical density profiles

Bosons	Fermions



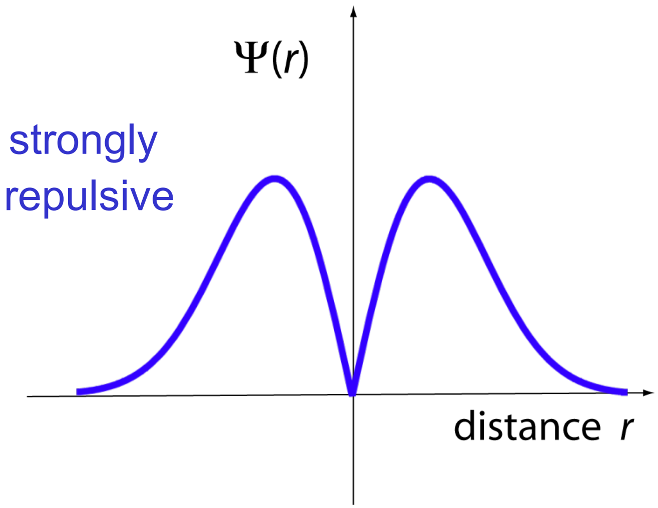


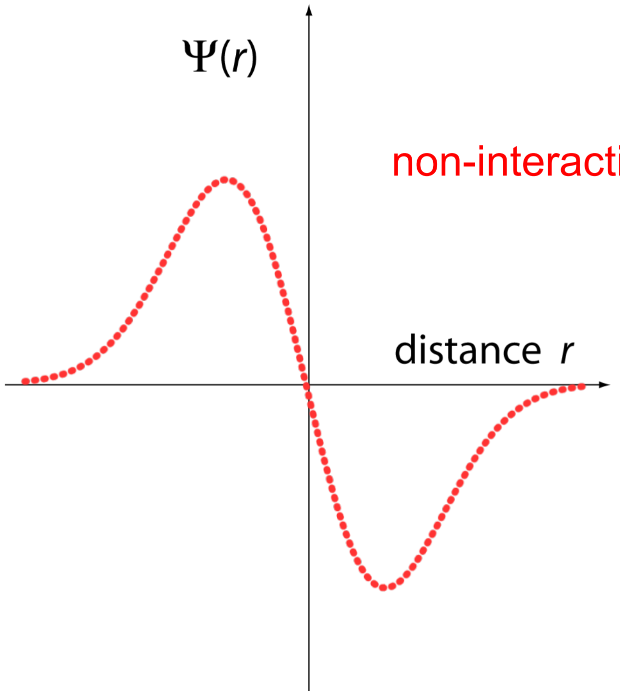
sketch: wave functions for two particles in harmonic trap

Bose-Fermi mapping

Identical density profiles

Bosons	Fermions





sketch: wave functions for two particles in harmonic trap

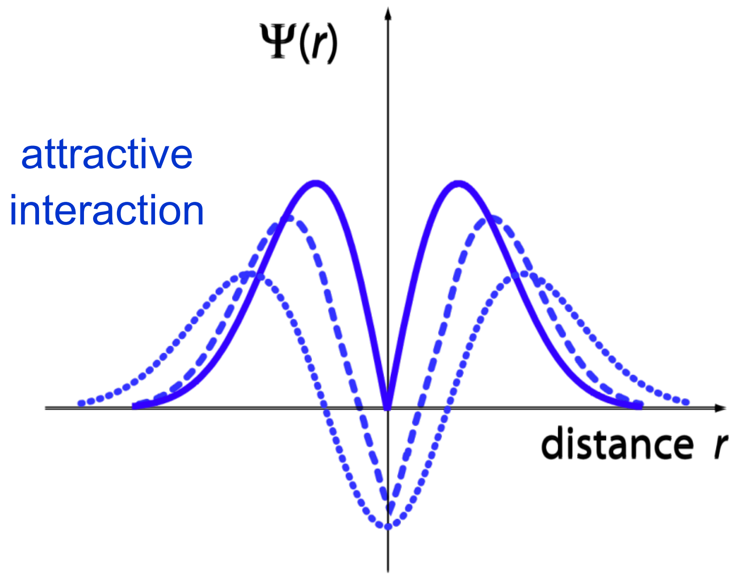
Extended Bose-Fermi mapping

Bose-Fermi mapping:

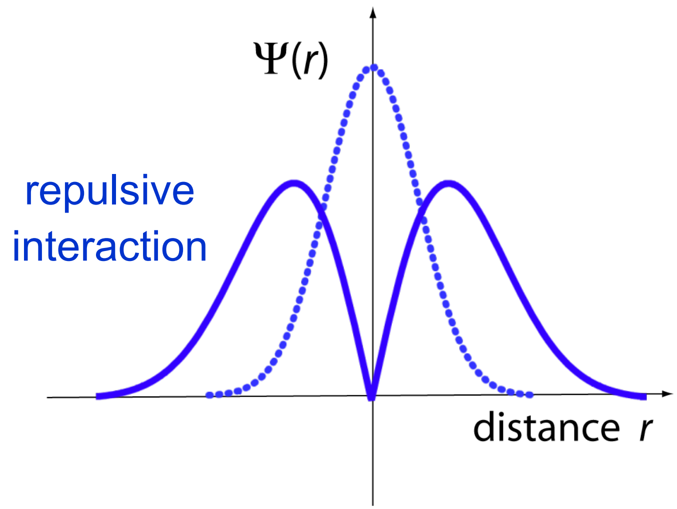
Astrakharchik *et al.*,
PRL **95** 190407 (2005)

Excited Bosons with attractive interactions and ground state Bosons with repulsive interactions show the same density distribution.

Bosons, excited



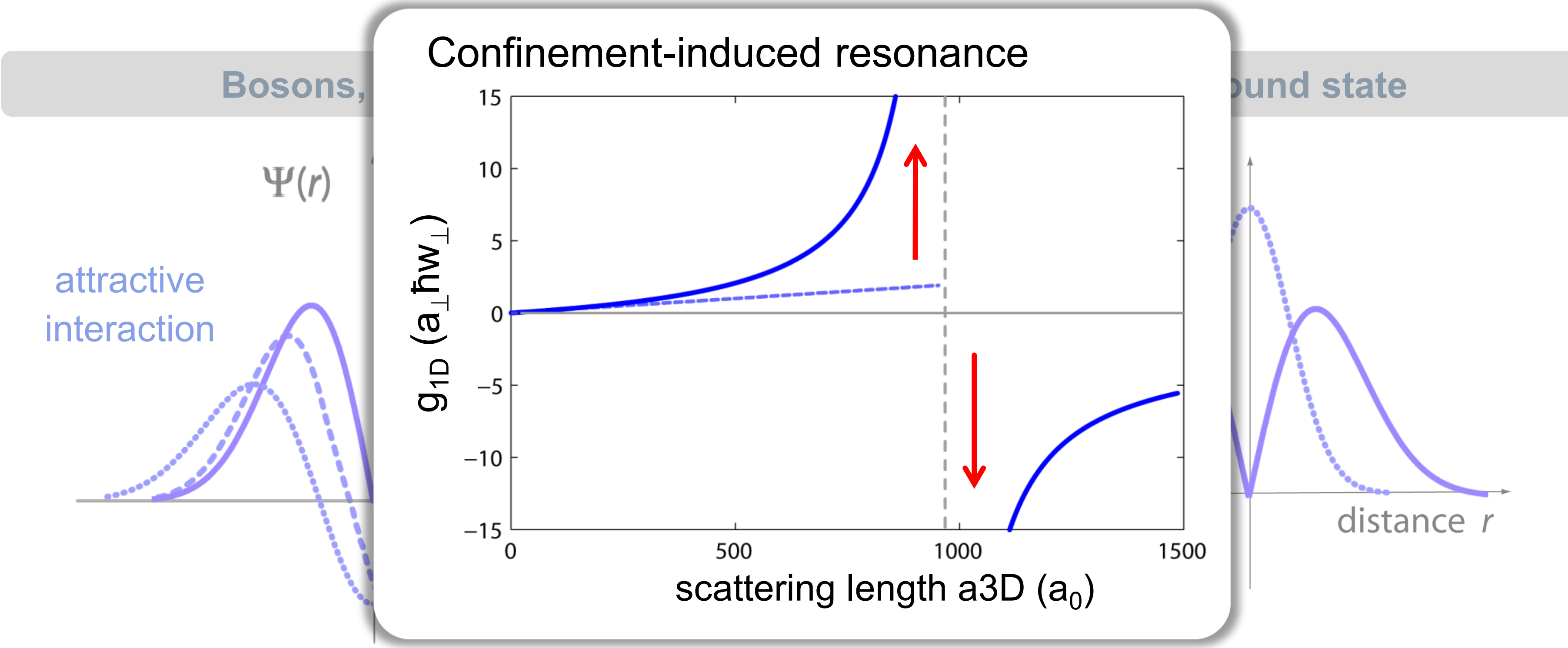
Bosons, ground state



sketch: wave functions for two particles in harmonic trap

Extended Bose-Fermi mapping

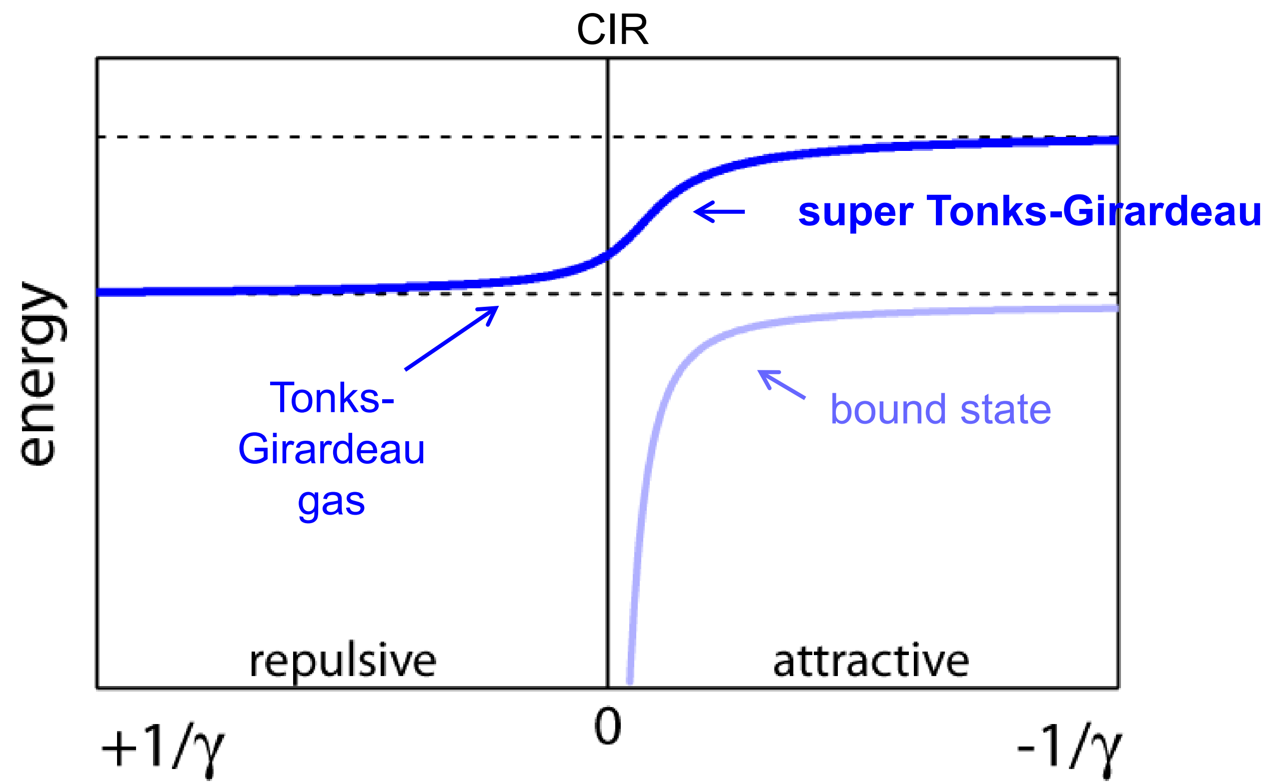
Matching wave functions on both sides of the confinement-induced resonance



sketch: wave functions for two particles in harmonic trap

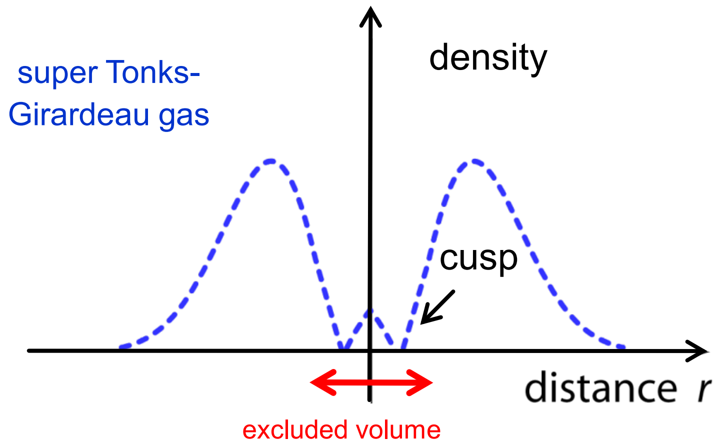
Super Tonks-Girardeau gas

Energy levels at the confinement-induced resonance



Properties of the super Tonks-Girardeau gas

Density profile



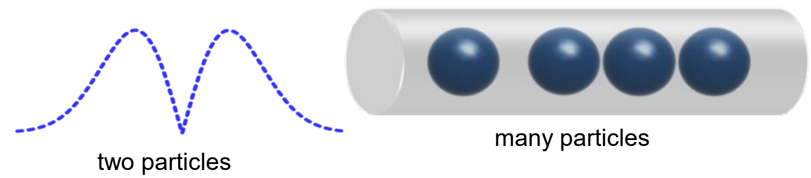
sketch: density for two particles in a harmonic trap

excited bosons,
+ attractive
interactions

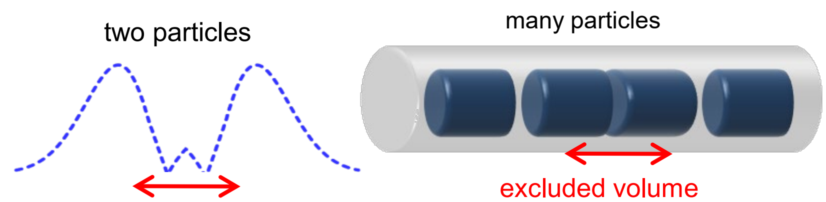
⇔

fermions,
+ repulsive
interactions

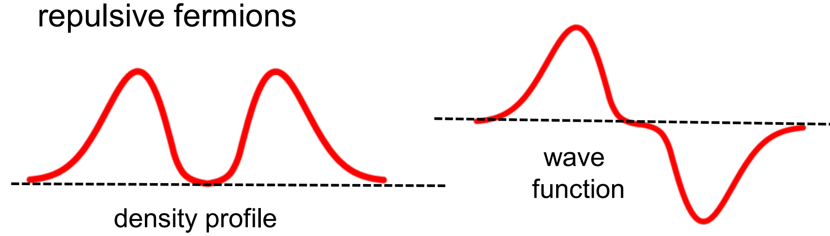
Tonks-Girardeau gas **(hard spheres)**



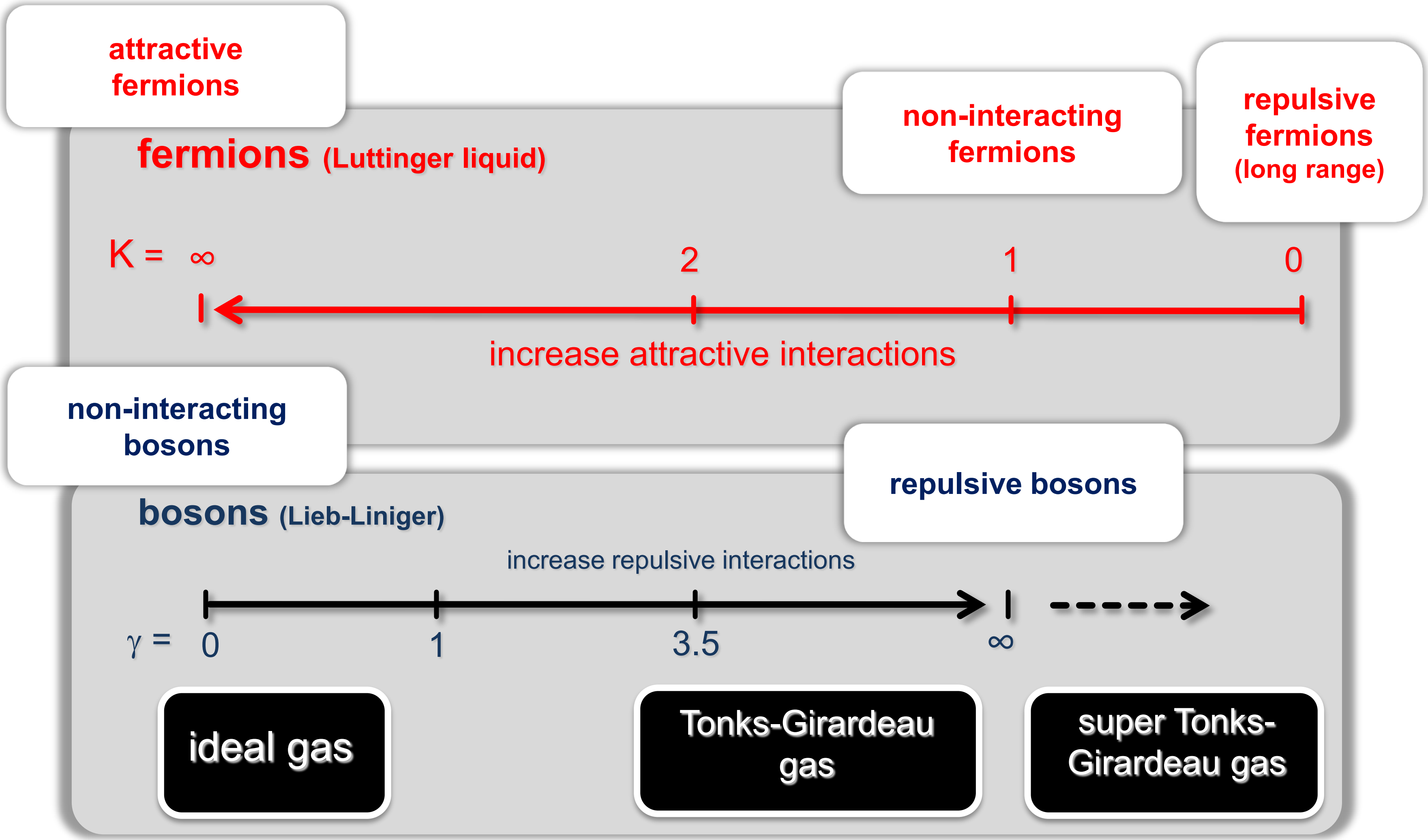
super Tonks-Girardeau gas (**hard rods**)



repulsive fermions



Interaction regimes of 1D quantum gases



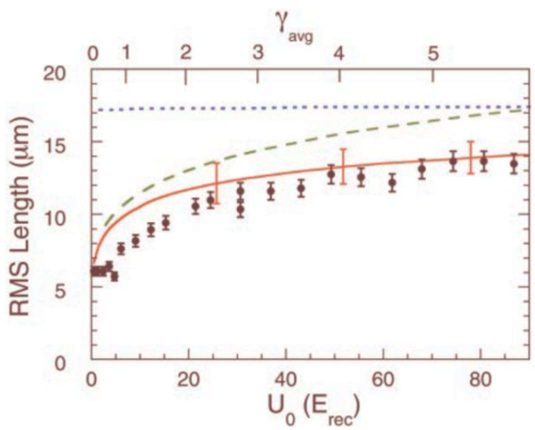
Realization of a Tonks-Giradeau gas

increase confinement strength

- γ depends on the density and confinement strength

$$\gamma = \frac{m g_{1D}}{\hbar^2 n} \sim \frac{\omega_\perp}{n}$$

T. Kinoshita *et al.*, Science **305,** 1125 (2004)

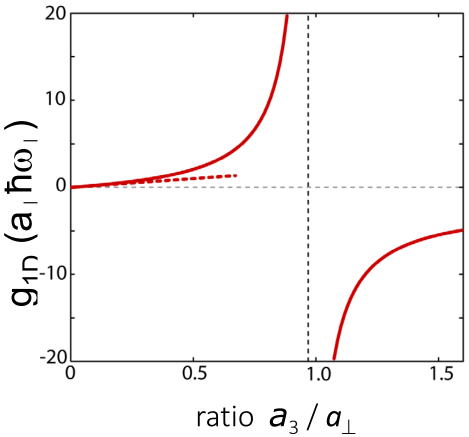


tune interactions with scattering resonance

- γ depends on g_{1D}

$$\gamma = \frac{m g_{1D}}{\hbar^2 n}$$

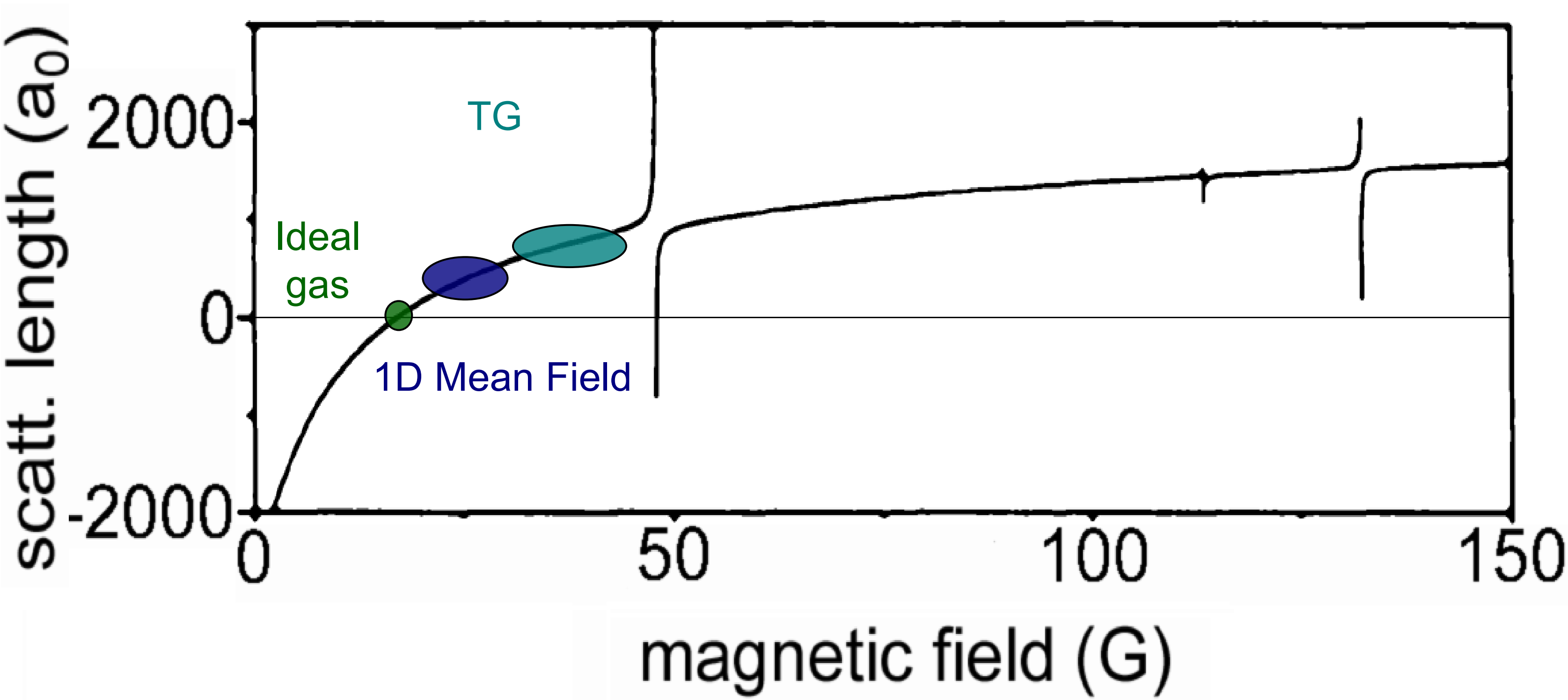
Innsbruck group,
Science **325,** 1224 (2009)



other approaches: B. Paredes *et al.*, Nature **429**, 277 (2004)
N. Syassen *et al.*, Science **320**, 1329 (2008)

Realization of a Tonks-Giradeau gas

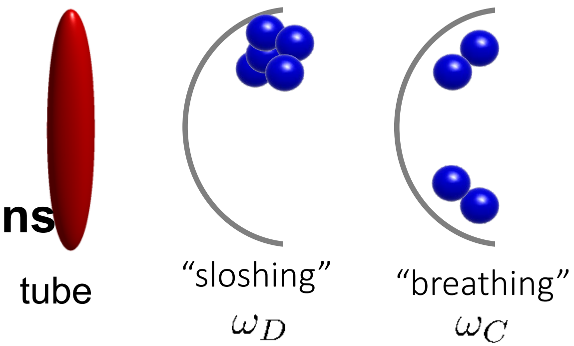
magnetic Feshbach resonances Cesium 133, F = 3, m_F = 3



Experimental results

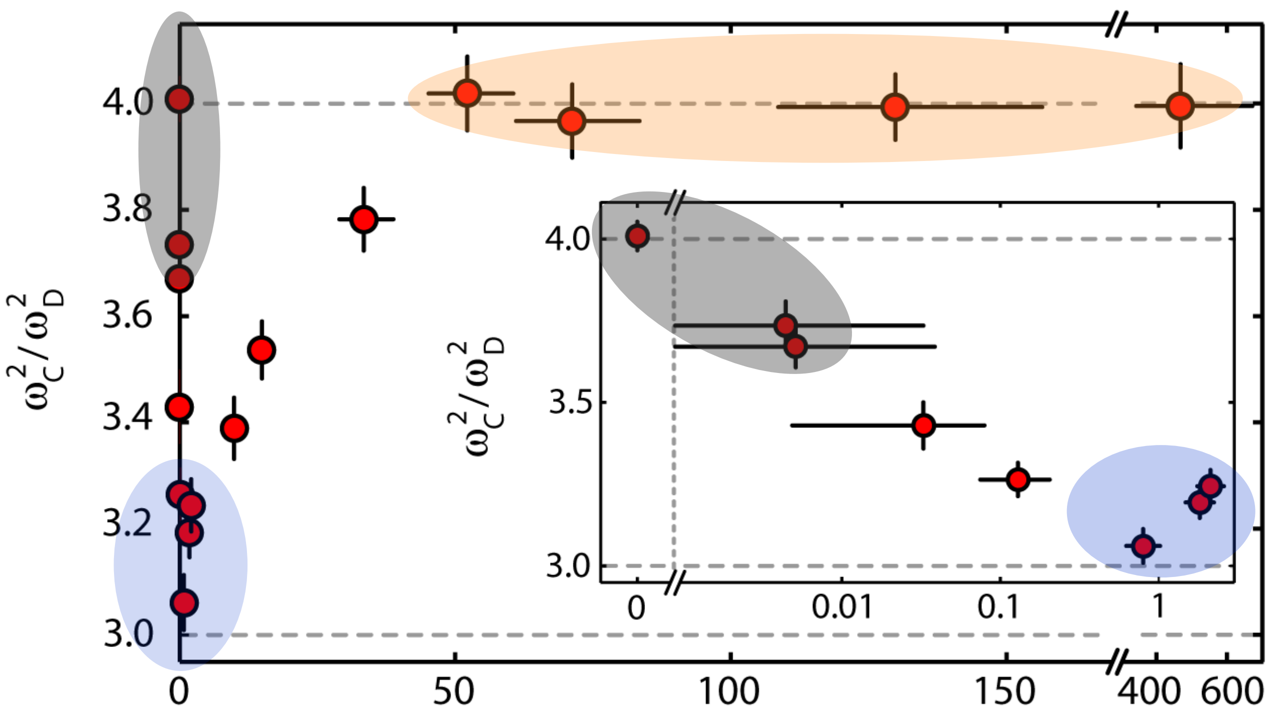
experimental probe:

collective oscillations



regimes	$(\omega_C/\omega_D)^2$
thermal / non-interacting	4
1D mean field	3
Tonks Girardeau	4

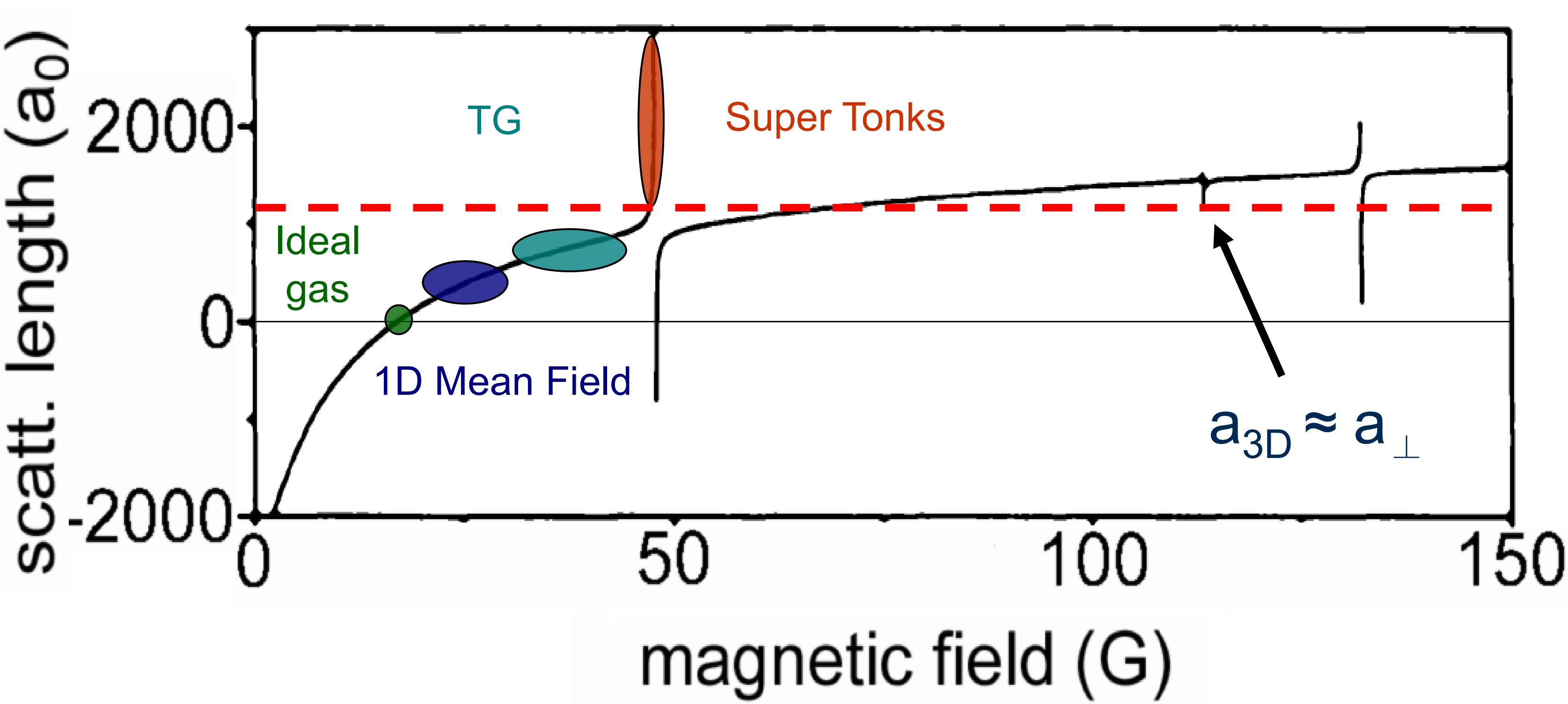
C. Menotti, S. Stringari, PRA **66** 043610 (2002)



interaction parameter $\gamma = \frac{E_{int}}{E_{kin}}$

Experimental realization

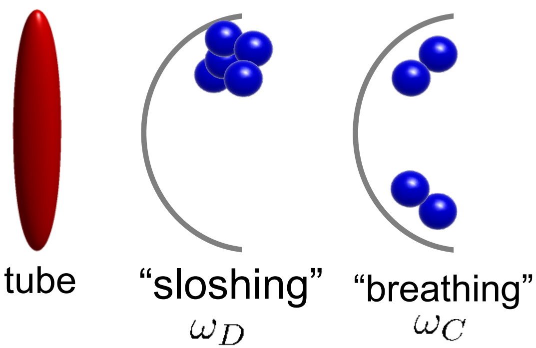
magnetic Feshbach resonances Cesium 133, F = 3, m_F = 3



Observation of the sTG gas

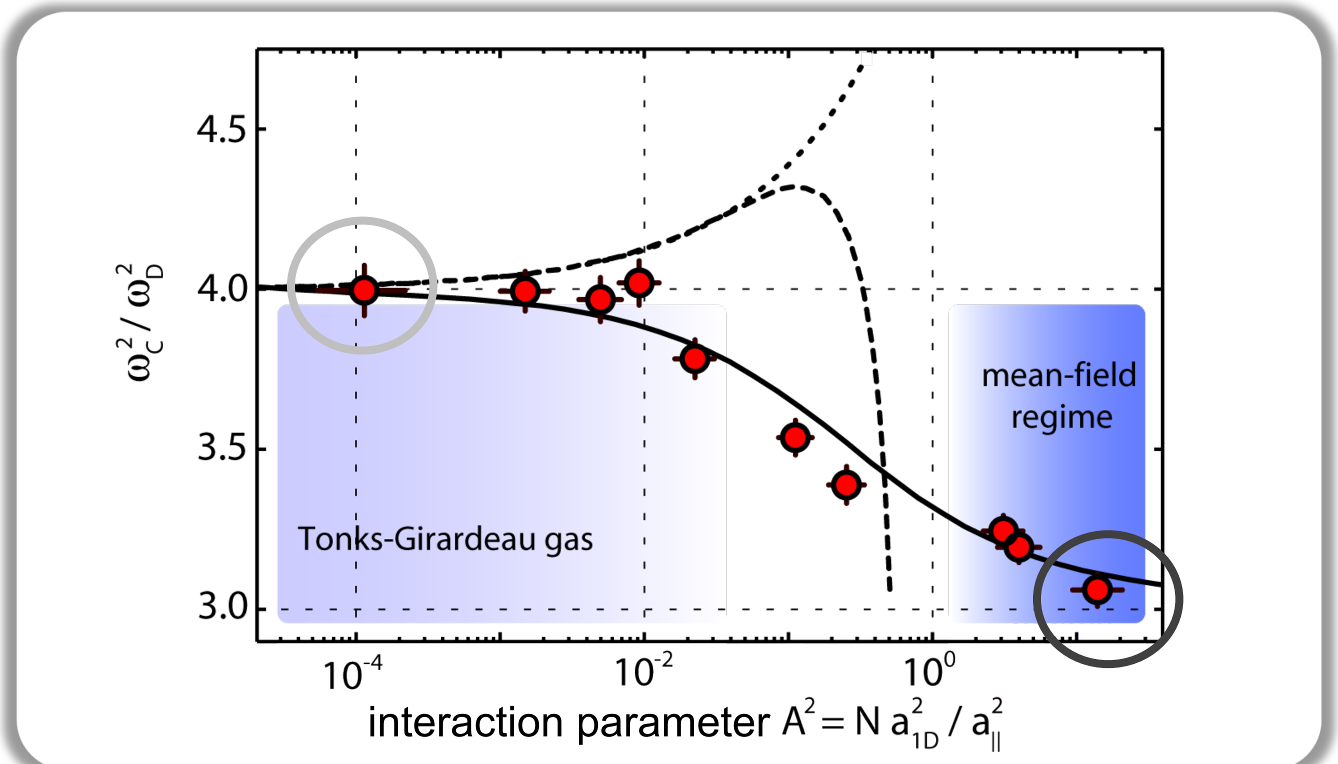
Collective oscillations

the oscillation frequency depends on the interaction regime.



interaction regimes	$(\omega_C/\omega_D)^2$
1D mean field	3
Tonks-Girardeau gas	4

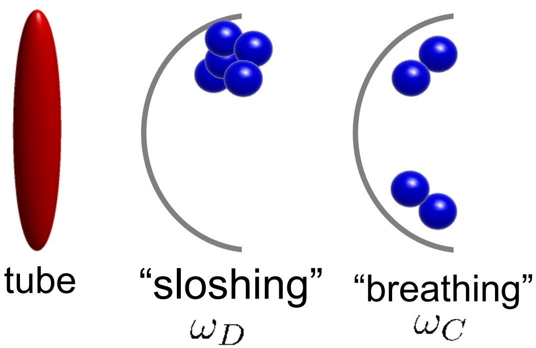
C. Menotti, S. Stringari, PRA **66** 043610 (2002)



Observation of the sTG gas

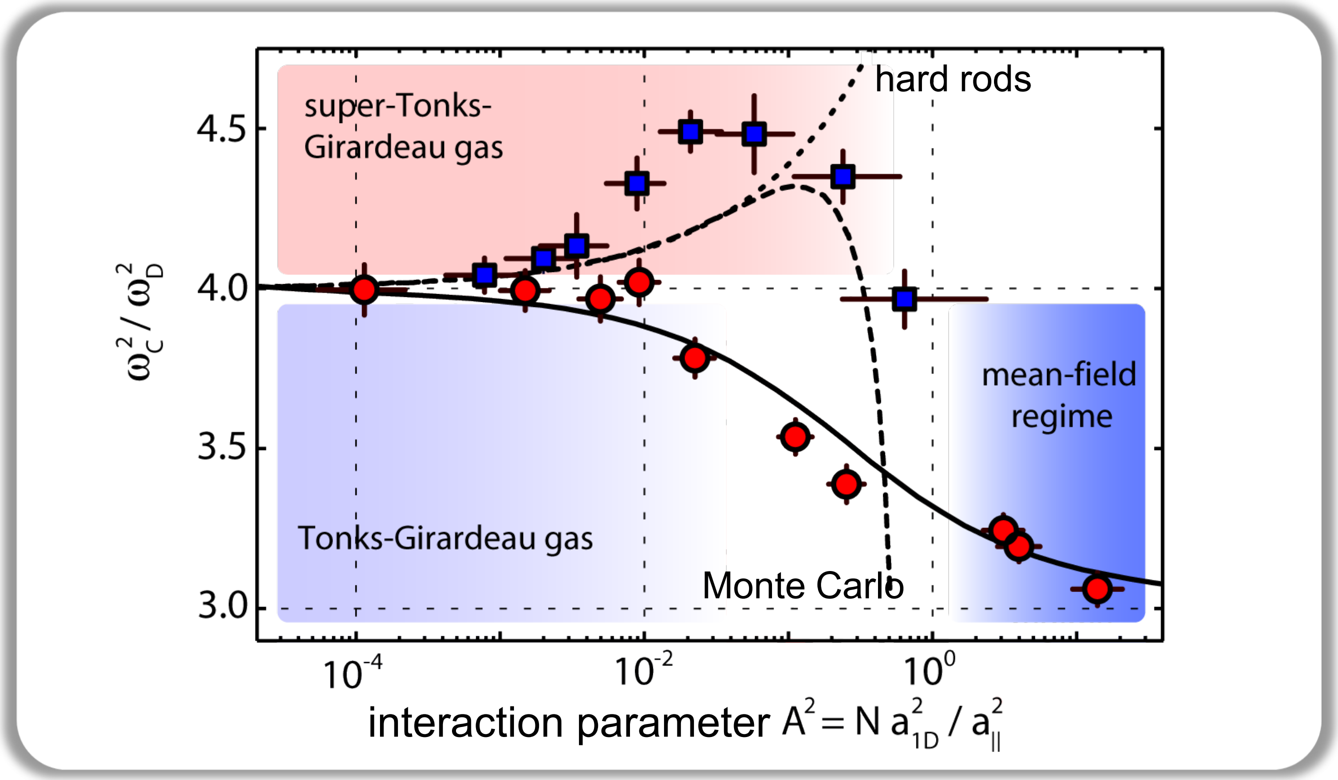
Collective oscillations

the oscillation frequency depends on the interaction regime.



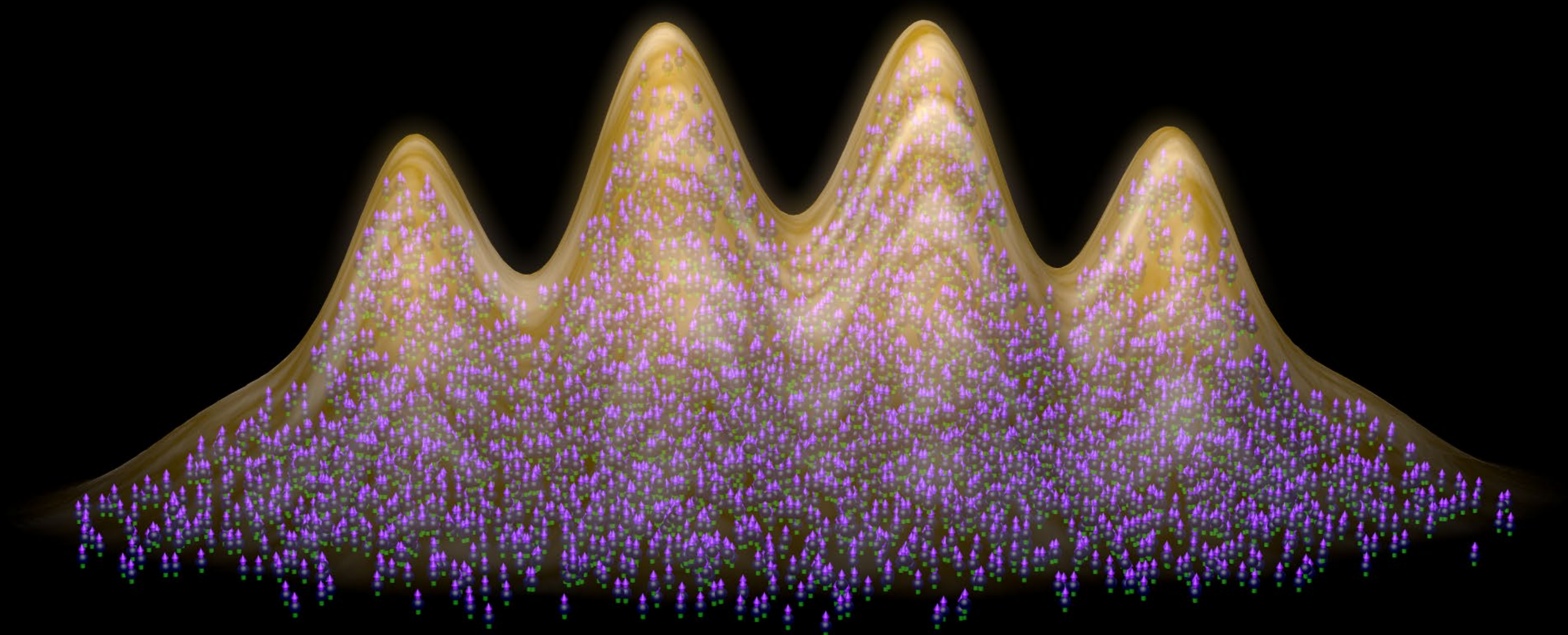
interaction regimes	$(\omega_C/\omega_D)^2$
1D mean field	3
Tonks-Girardeau gas	4
Super-Tonks Girardeau	> 4

C. Menotti, S. Stringari, PRA **66** 043610 (2002)



E. Haller *et al.*, Science **325** 1224 (2009)

Thanks for your attention!



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